200 CÂU TÍCH PHÂN

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Dạng 1: Tách phân thức

Câu 1. $I = \int_1^2 \frac{x^2}{x^2 - 7x + 12} dx$

• $I = \int_1^2 \left(1 + \frac{16}{x-4} - \frac{9}{x-3}\right) dx = \left(x + 16\ln|x-4| - 9\ln|x-3|\right)\Big|_1^2 = 1 + 25\ln 2 - 16\ln 3$.

Câu 2. $I = \int_1^2 \frac{dx}{x^5 + x^3}$

• *Ta có:* $\frac{1}{x^3(x^2+1)} = -\frac{1}{x} + \frac{1}{x^3} + \frac{x}{x^2+1}$

$\Rightarrow I = \left[-\ln|x| - \frac{1}{2x^2} + \frac{1}{2}\ln(x^2+1)\right]\Big|_1^2 = -\frac{3}{2}\ln 2 + \frac{1}{2}\ln 5 + \frac{3}{8}$

Câu 3. $I = \int_4^5 \frac{3x^2+1}{x^3 - 2x^2 - 5x + 6} dx$ • $I = -\frac{2}{3}\ln\frac{4}{3} + \frac{13}{15}\ln\frac{7}{6} + \frac{14}{5}\ln 2$

Dạng 2: Đổi biến số

Câu 4. $I = \int \frac{(x-1)^2}{(2x+1)^4} dx$ • Ta có: $f(x) = \frac{1}{3}\cdot\left(\frac{x-1}{2x+1}\right)^2 \cdot \left(\frac{x-1}{2x+1}\right)' \Rightarrow I = \frac{1}{9}\left(\frac{x-1}{2x+1}\right)^3 + C$

Câu 5. $I = \int_0^1 \frac{(7x-1)^{99}}{(2x+1)^{101}} dx$

• $I = \int_0^1 \left(\frac{7x-1}{2x+1}\right)^{99} \frac{dx}{(2x+1)^2} = \frac{1}{9}\int_0^1 \left(\frac{7x-1}{2x+1}\right)^{99} d\left(\frac{7x-1}{2x+1}\right)$

$= \frac{1}{9}\cdot\frac{1}{100}\left(\frac{7x-1}{2x+1}\right)^{100}\Big|_0^1 = \frac{1}{900}\left[2^{100} - 1\right]$

Câu 6. $I = \int_0^1 \frac{5x}{(x^2+4)^2} dx$ • *Đặt* $t = x^2 + 4 \Rightarrow I = \frac{1}{8}$

Câu 7. $I = \int_1^{\sqrt[4]{3}} \frac{1}{x(x^4+1)} dx$ • *Đặt* $t = x^2 \Rightarrow I = \frac{1}{2}\int_1^{\sqrt{3}} \left(\frac{1}{t} - \frac{t}{t^2+1}\right) dt = \frac{1}{4}\ln\frac{3}{2}$

Câu 8. $I = \int_1^{\sqrt{3}} \frac{dx}{x^6(1+x^2)}$

• *Đặt :* $x=\frac{1}{t} \Rightarrow I=-\int_{1}^{\frac{\sqrt{3}}{3}}\frac{t^6}{t^2+1}dt=\int_{\frac{\sqrt{3}}{3}}^{1}\left(t^4-t^2+1-\frac{1}{t^2+1}\right)dt = \frac{117-41\sqrt{3}}{135}+\frac{\pi}{12}$

Câu 9. $I=\int_1^2\frac{dx}{x.(x^{10}+1)^2}$

• $I=\int_1^2\frac{x^4.dx}{x^5.(x^{10}+1)^2}$. *Đặt* $t=x^5 \Rightarrow I=\frac{1}{5}\int_1^{32}\frac{dt}{t(t^2+1)^2}$

Câu 10. $I=\int_0^1\frac{x^7}{(1+x^2)^5}dx$

• *Đặt* $t=1+x^2 \Rightarrow dt=2xdx \Rightarrow I=\frac{1}{2}\int_1^2\frac{(t-1)^3}{t^5}dt=\frac{1}{4}.\frac{1}{2^5}$

Câu 11. $I=\int_1^2\frac{1-x^7}{x(1+x^7)}dx$

• $I=\int_1^2\frac{(1-x^7).x^6}{x^7.(1+x^7)}dx$. *Đặt* $t=x^7 \Rightarrow I=\frac{1}{7}\int_1^{128}\frac{1-t}{t(1+t)}dt$

Câu 12. $I=\int_1^2\frac{x^{2001}}{(1+x^2)^{1002}}.dx$

• $I=\int_1^2\frac{x^{2004}}{x^3(1+x^2)^{1002}}.dx=\int_1^2\frac{1}{x^3\left(\frac{1}{x^2}+1\right)^{1002}}.dx$. *Đặt* $t=\frac{1}{x^2}+1 \Rightarrow dt=-\frac{2}{x^3}dx$.

<u>*Cách 2*</u>*: Ta có:* $I=\frac{1}{2}\int_0^1\frac{x^{2000}.2xdx}{(1+x^2)^{2000}(1+x^2)^2}$. *Đặt* $t=1+x^2 \Rightarrow dt=2xdx$

$\Rightarrow I=\frac{1}{2}\int_1^2\frac{(t-1)^{1000}}{t^{1000}t^2}dt=\frac{1}{2}\int_1^2\left(1-\frac{1}{t}\right)^{1000}d\left(1-\frac{1}{t}\right)=\frac{1}{2002.2^{1001}}$

Câu 13. $I=\int_0^1 x^5(1-x^3)^6dx$

• *Đặt* $t=1-x^3 \Rightarrow dt=-3x^2dx \Rightarrow dx=\frac{-dt}{3x^2} \Rightarrow I=\frac{1}{3}\int_0^1 t^6(1-t)dt=\frac{1}{3}\left(\frac{t^7}{7}-\frac{t^8}{8}\right)=\frac{1}{168}$

Câu 14. $I=\int_0^1\frac{xdx}{(x+1)^3}$

• *Ta có:* $\frac{x}{(x+1)^3}=\frac{x+1-1}{(x+1)^3}=(x+1)^{-2}-(x+1)^{-3} \Rightarrow I=\int_0^1\left[(x+1)^{-2}-(x+1)^{-3}\right]dx=\frac{1}{8}$

Câu 15. $I=\int_1^2\frac{1+x^2}{1+x^4}dx$

• *Ta có:* $\frac{1+x^2}{1+x^4}=\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}$. *Đặt* $t=x-\frac{1}{x} \Rightarrow dt=\left(1+\frac{1}{x^2}\right)dx$

$\Rightarrow I=\int_1^{\frac{3}{2}}\frac{dt}{t^2-2}=\frac{1}{2\sqrt{2}}\int_1^{\frac{3}{2}}\left(\frac{1}{t-\sqrt{2}}-\frac{1}{t+\sqrt{2}}\right)dt=\frac{1}{2\sqrt{2}}.\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right|\Bigg|_1^{3/2}=\frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$

Câu 16. $I=\int_1^2 \frac{1-x^2}{1+x^4}dx$

• *Ta có:* $\frac{1-x^2}{1+x^4}=\frac{\frac{1}{x^2}-1}{x^2+\frac{1}{x^2}}$. *Đặt* $t=x+\frac{1}{x}\Rightarrow dt=\left(1-\frac{1}{x^2}\right)dx \Rightarrow I=-\int_2^{\frac{5}{2}}\frac{dt}{t^2+2}$.

Đặt $t=\sqrt{2}\tan u\Rightarrow dt=\sqrt{2}\frac{du}{\cos^2 u}$; $\tan u=2\Rightarrow u_1=\arctan 2$; $\tan u=\frac{5}{2}\Rightarrow u_2=\arctan\frac{5}{2}$

$\Rightarrow I=\frac{\sqrt{2}}{2}\int_{u_1}^{u_2}du=\frac{\sqrt{2}}{2}(u_2-u_1)=\frac{\sqrt{2}}{2}\left(\arctan\frac{5}{2}-\arctan 2\right)$

Câu 17. $I=\int_0^1 \frac{x^4+1}{x^6+1}dx$

• *Ta có:* $\frac{x^4+1}{x^6+1}=\frac{(x^4-x^2+1)+x^2}{x^6+1}=\frac{x^4-x^2+1}{(x^2+1)(x^4-x^2+1)}+\frac{x^2}{x^6+1}=\frac{1}{x^2+1}+\frac{x^2}{x^6+1}$

$\Rightarrow I=\int_0^1\frac{1}{x^2+1}dx+\frac{1}{3}\int_0^1\frac{d(x^3)}{(x^3)^2+1}dx=\frac{\pi}{4}+\frac{1}{3}\frac{\pi}{4}=\frac{\pi}{3}$

Câu 18. $I=\int_1^2\frac{1-x^2}{x+x^3}dx$ • *Ta có:* $I=\int_1^2\frac{\frac{1}{x^2}-1}{\frac{1}{x}+x}dx$. *Đặt* $t=x+\frac{1}{x}\Rightarrow I=\ln\frac{4}{5}$

Câu 19. $I=\int_0^1\frac{xdx}{x^4+x^2+1}$. • *Đặt* $t=x^2\Rightarrow I=\frac{1}{2}\int_0^1\frac{dt}{t^2+t+1}=\frac{1}{2}\int_0^1\frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{\pi}{6\sqrt{3}}$

Câu 20. $I=\int_1^{\frac{1+\sqrt{5}}{2}}\frac{x^2+1}{x^4-x^2+1}dx$

• *Ta có:* $\frac{x^2+1}{x^4-x^2+1}=\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1}$. *Đặt* $t=x-\frac{1}{x}\Rightarrow dt=\left(1+\frac{1}{x^2}\right)dx$

$\Rightarrow I=\int_0^1\frac{dt}{t^2+1}$. *Đặt* $t=\tan u\Rightarrow dt=\frac{du}{\cos^2 u}\Rightarrow I=\int_0^{\frac{\pi}{4}}du=\frac{\pi}{4}$

Câu 21. $I=\int_0^{\frac{\sqrt{3}}{3}}\frac{x^2}{x^4-1}dx$

• $I=\int_0^{\frac{\sqrt{3}}{3}}\frac{x^2}{(x^2-1)(x^2+1)}dx=\frac{1}{2}\int_0^{\frac{\sqrt{3}}{3}}\left(\frac{1}{x^2-1}+\frac{1}{x^2+1}\right)dx=\frac{1}{4}\ln(2-\sqrt{3})+\frac{\pi}{12}$

TP2: TÍCH PHÂN HÀM SỐ VÔ TỈ

Dạng 1: Đổi biến số dạng 1

Câu 1. $I = \int \frac{x}{3x + \sqrt{9x^2 - 1}} dx$

• $I = \int \frac{x}{3x + \sqrt{9x^2 - 1}} dx = \int x(3x - \sqrt{9x^2 - 1})dx = \int 3x^2 dx - \int x\sqrt{9x^2 - 1}dx$

$+ I_1 = \int 3x^2 dx = x^3 + C_1$ $\quad + I_2 = \int x\sqrt{9x^2 - 1}dx = \frac{1}{18}\int \sqrt{9x^2 - 1}\, d(9x^2 - 1) = \frac{1}{27}(9x^2 - 1)^{\frac{3}{2}} + C_2$

$\Rightarrow I = \frac{1}{27}(9x^2 - 1)^{\frac{3}{2}} + x^3 + C$

Câu 2. $I = \int \frac{x^2 + \sqrt{x}}{\sqrt{1 + x\sqrt{x}}} dx$

• $\int \frac{x^2 + \sqrt{x}}{\sqrt{1 + x\sqrt{x}}} dx = \int \frac{x^2}{\sqrt{1 + x\sqrt{x}}} dx + \int \frac{\sqrt{x}}{\sqrt{1 + x\sqrt{x}}} dx$.

$+ I_1 = \int \frac{x^2}{\sqrt{1 + x\sqrt{x}}} dx$. *Đặt* $t = \sqrt{1 + x\sqrt{x}} \Leftrightarrow t^2 - 1 = x\sqrt{x} \Leftrightarrow x^3 = (t^2 - 1)^2 \Leftrightarrow x^2 dx = \frac{4}{3}t(t^2 - 1)dt$

$\Rightarrow \int \frac{4}{3}(t^2 - 1)dt = \frac{4}{9}t^3 - \frac{4}{3}t + C = \frac{4}{9}\left(\sqrt{1 + x\sqrt{x}}\right)^3 - \frac{4}{3}\sqrt{1 + x\sqrt{x}} + C_1$

$+ I_2 = \int \frac{\sqrt{x}}{\sqrt{1 + x\sqrt{x}}} dx = \frac{2}{3}\int \frac{d(1 + x\sqrt{x})}{\sqrt{1 + x\sqrt{x}}} = \frac{4}{3}\sqrt{1 + x\sqrt{x}} + C_2$

Vậy: $I = \frac{4}{9}\left(\sqrt{1 + x\sqrt{x}}\right)^3 + C$

Câu 3. $I = \int_0^4 \frac{\sqrt{2x + 1}}{1 + \sqrt{2x + 1}} dx$ • *Đặt* $t = \sqrt{2x + 1}$. $I = \int_1^3 \frac{t^2}{1 + t} dt = 2 + \ln 2$.

Câu 4. $I = \int_2^6 \frac{dx}{2x + 1 + \sqrt{4x + 1}}$ • *Đặt* $t = \sqrt{4x + 1}$. $I = \ln\frac{3}{2} - \frac{1}{12}$

Câu 5. $I = \int_0^1 x^3\sqrt{1 - x^2}dx$ • *Đặt:* $t = \sqrt{1 - x^2} \Rightarrow I = \int_0^1 \left(t^2 - t^4\right)dt = \frac{2}{15}$.

Câu 6. $I = \int_0^1 \frac{1 + x}{1 + \sqrt{x}} dx$

• *Đặt* $t = \sqrt{x} \Rightarrow dx = 2t.dt$. $I = 2\int_0^1 \frac{t^3 + t}{t + 1} dt = 2\int_0^1 \left(t^2 - t + 2 - \frac{2}{1 + t}\right)dt = \frac{11}{3} - 4\ln 2$.

Câu 7. $I = \int_0^3 \frac{x - 3}{3\sqrt{x + 1} + x + 3} dx$

• *Đặt* $t=\sqrt{x+1}\Rightarrow 2tdu=dx\ \Rightarrow I=\int_1^2\frac{2t^3-8t}{t^2+3t+2}dt=\int_1^2(2t-6)dt+6\int_1^2\frac{1}{t+1}dt=-3+6\ln\frac{3}{2}$

Câu 8. $I=\int_{-1}^{0}x\sqrt[3]{x+1}dx$

• *Đặt* $t=\sqrt[3]{x+1}\Rightarrow t^3=x+1\Rightarrow dx=3t^2dt\Rightarrow I=\int_0^1 3(t^3-1)dt=3\left(\frac{t^7}{7}-\frac{t^4}{4}\right)\Big|_0^1=-\frac{9}{28}$

Câu 9. $I=\int_1^5\frac{x^2+1}{x\sqrt{3x+1}}dx$

• *Đặt* $t=\sqrt{3x+1}\ \Rightarrow dx=\frac{2tdt}{3}\ \Rightarrow I=\int_2^4\frac{\left(\frac{t^2-1}{3}\right)^2+1}{\frac{t^2-1}{3}.t}.\frac{2tdt}{3}\ =\frac{2}{9}\int_2^4(t^2-1)dt+2\int_2^4\frac{dt}{t^2-1}$

$=\frac{2}{9}\left(\frac{1}{3}t^3-t\right)\Big|_2^4+\ln\left|\frac{t-1}{t+1}\right|\Big|_2^4=\frac{100}{27}+\ln\frac{9}{5}.$

Câu 10. $I=\int_0^3\frac{2x^2+x-1}{\sqrt{x+1}}dx$

• *Đặt* $\sqrt{x+1}=t\Leftrightarrow x=t^2-1\ \Rightarrow dx=2tdt$

$\Rightarrow I=\int_1^2\frac{2(t^2-1)^2+(t^2-1)-1}{t}2tdt\ =2\int_1^2(2t^4-3t^2)dt=\left(\frac{4t^5}{5}-2t^3\right)\Big|_1^2=\frac{54}{5}$

Câu 11. $I=2\int_0^1\frac{x^2dx}{(x+1)\sqrt{x+1}}$

• *Đặt* $t=\sqrt{x+1}\Rightarrow t^2=x+1\Rightarrow 2tdt=dx$

$\Rightarrow I=\int_1^{\sqrt{2}}\frac{(t^2-1)^2}{t^3}.2tdt=2\int_1^{\sqrt{2}}\left(t-\frac{1}{t}\right)^2dt=2\left(\frac{t^3}{3}-2t-\frac{1}{t}\right)\Big|_1^{\sqrt{2}}=\frac{16-11\sqrt{2}}{3}$

Câu 12. $I=\int_0^4\frac{x+1}{\left(1+\sqrt{1+2x}\right)^2}dx$

• *Đặt* $t=1+\sqrt{1+2x}\Rightarrow dt=\frac{dx}{\sqrt{1+2x}}\Rightarrow dx=(t-1)dt$ *và* $x=\frac{t^2-2t}{2}$

Ta có: $I=\frac{1}{2}\int_2^4\frac{(t^2-2t+2)(t-1)}{t^2}dt=\frac{1}{2}\int_2^4\frac{t^3-3t^2+4t-2}{t^2}dt=\frac{1}{2}\int_2^4\left(t-3+\frac{4}{t}-\frac{2}{t^2}\right)dt$

$=\frac{1}{2}\left(\frac{t^2}{2}-3t+4\ln|t|+\frac{2}{t}\right)\Big|=2\ln 2-\frac{1}{4}$

Câu 13. $I=\int_{\sqrt{3}}^{\sqrt{8}}\frac{x-1}{\sqrt{x^2+1}}dx$

• $I = \int_{\sqrt{3}}^{\sqrt{8}} \left(\frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}} \right) dx = \left[\sqrt{x^2+1} - \ln\left(x + \sqrt{x^2+1}\right) \right] \Big|_{\sqrt{3}}^{\sqrt{8}} = 1 + \ln\left(\sqrt{3}+2\right) - \ln\left(\sqrt{8}+3\right)$

Câu 14. $I = \int_0^1 (x-1)^3 \sqrt{2x - x^2}\, dx$

• $I = \int_0^1 (x-1)^3 \sqrt{2x-x^2}\, dx = \int_0^1 (x^2 - 2x + 1)\sqrt{2x-x^2}\,(x-1)dx$. *Đặt* $t = \sqrt{2x - x^2} \Rightarrow I = -\frac{2}{15}$.

Câu 15. $I = \int_0^2 \frac{2x^3 - 3x^2 + x}{\sqrt{x^2 - x + 1}} dx$

• $I = \int_0^2 \frac{(x^2 - x)(2x-1)}{\sqrt{x^2 - x + 1}} dx$. *Đặt* $t = \sqrt{x^2 - x + 1} \Rightarrow I = 2\int_1^{\sqrt{3}} (t^2 - 1)dt = \frac{4}{3}$.

Câu 16. $I = \int_0^2 \frac{x^3 dx}{\sqrt[3]{4 + x^2}}$

• *Đặt* $t = \sqrt[3]{4 + x^2} \Rightarrow x^2 = t^3 - 4 \Rightarrow 2xdx = 3t^2 dt \Rightarrow I = \frac{3}{2}\int_{\sqrt[3]{4}}^{2} (t^4 - 4t)dt = -\frac{3}{2}\left(\frac{8}{5} + 4\sqrt[3]{2}\right)$

Câu 17. $I = \int_{-1}^{1} \frac{dx}{1 + x + \sqrt{1 + x^2}}$

• *Ta có:* $I = \int_{-1}^{1} \frac{1 + x - \sqrt{1+x^2}}{(1+x)^2 - (1+x^2)} dx = \int_{-1}^{1} \frac{1 + x - \sqrt{1+x^2}}{2x} dx = \frac{1}{2}\int_{-1}^{1}\left(\frac{1}{x} + 1\right)dx - \int_{-1}^{1} \frac{\sqrt{1+x^2}}{2x} dx$

\+ $I_1 = \frac{1}{2}\int_{-1}^{1}\left(\frac{1}{x}+1\right)dx = \frac{1}{2}\left[\ln|x| + x\right]\Big|_{-1}^{1} = 1$

\+ $I_2 = \int_{-1}^{1} \frac{\sqrt{1+x^2}}{2x} dx$. *Đặt* $t = \sqrt{1+x^2} \Rightarrow t^2 = 1 + x^2 \Rightarrow 2tdt = 2xdx \Rightarrow I_2 = \int_{\sqrt{2}}^{\sqrt{2}} \frac{t^2 dt}{2(t^2-1)} = 0$

Vậy: $I = 1$.

Cách 2: Đặt $t = x + \sqrt{x^2 + 1}$.

Câu 18. $I = \int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$ • *Ta có:* $I = \int_{\frac{1}{3}}^{1} \left(\frac{1}{x^2} - 1\right)^{\frac{1}{3}} . \frac{1}{x^3} dx$. *Đặt* $t = \frac{1}{x^2} - 1 \Rightarrow I = 6$.

Câu 19. $I = \int_1^2 \frac{\sqrt{4 - x^2}}{x} dx$

• *Ta có:* $I = \int_1^2 \frac{\sqrt{4 - x^2}}{x^2} xdx$. *Đặt* $t = \sqrt{4 - x^2} \Rightarrow t^2 = 4 - x^2 \Rightarrow tdt = -xdx$

$\Rightarrow I = \int_{\sqrt{3}}^{0} \frac{t(-tdt)}{4 - t^2} = \int_{\sqrt{3}}^{0} \frac{t^2}{t^2 - 4} dt = \int_{\sqrt{3}}^{0} \left(1 + \frac{4}{t^2 - 4}\right)dt = \left(t + \ln\left|\frac{t-2}{t+2}\right|\right)\Big|_{\sqrt{3}}^{0} = -\left(\sqrt{3} + \ln\left|\frac{2-\sqrt{3}}{2+\sqrt{3}}\right|\right)$

Câu 20. $I=\int_{2}^{2\sqrt{5}}\frac{x}{(x^2+1)\sqrt{x^2+5}}dx$ • *Đặt* $t=\sqrt{x^2+5}\Rightarrow I=\int_{3}^{5}\frac{dt}{t^2-4}=\frac{1}{4}\ln\frac{15}{7}$.

Câu 21. $I=\int_{1}^{27}\frac{\sqrt{x}-2}{x+\sqrt[3]{x^2}}dx$

• *Đặt* $t=\sqrt[6]{x}\Rightarrow I=5\int_{1}^{\sqrt{3}}\frac{t^3-2}{t(t^2+1)}dt=5\int_{1}^{\sqrt{3}}\left[1-\frac{2}{t}+\frac{2t}{t^2+1}-\frac{1}{t^2+1}\right]dt=5\left(\sqrt{3}-1+\ln\frac{2}{3}\right)-\frac{5\pi}{12}$

Câu 22. $I=\int_{0}^{1}\frac{1}{\sqrt{x^2+x+1}}dx$

• *Đặt* $t=x+\sqrt{x^2+x+1}\Rightarrow I=\int_{1}^{1+\sqrt{3}}\frac{2dt}{2t+1}=\ln(2t+1)\Big|_{1}^{1+\sqrt{3}}=\ln\frac{3+2\sqrt{3}}{3}$

Câu 23. $I=\int_{0}^{3}\frac{x^2}{(1+\sqrt{1+x})^2(2+\sqrt{1+x})^2}dx$

• *Đặt* $2+\sqrt{1+x}=t\Rightarrow I=\int_{3}^{4}\left(2t-16+\frac{42}{t}-\frac{36}{t^2}\right)dt=-12+42\ln\frac{4}{3}$

Câu 24. $I=\int_{0}^{3}\frac{x^2}{2(x+1)+2\sqrt{x+1}+x\sqrt{x+1}}dx$

• *Đặt* $t=\sqrt{x+1}\Rightarrow I=\int_{1}^{2}\frac{2t(t^2-1)^2dt}{t(t+1)^2}=2\int_{1}^{2}(t-1)^2dt=\frac{2}{3}(t-1)^3\Big|_{1}^{2}=\frac{2}{3}$

Câu 25. $I=\int_{1}^{2\sqrt{2}}\frac{\sqrt[3]{x-x^3}+2011x}{x^4}dx$

• *Ta có:* $I=\int_{1}^{2\sqrt{2}}\frac{\sqrt[3]{\frac{1}{x^2}-1}}{x^3}dx+\int_{1}^{2\sqrt{2}}\frac{2011}{x^3}dx=M+N$

$M=\int_{1}^{2\sqrt{2}}\frac{\sqrt[3]{\frac{1}{x^2}-1}}{x^3}dx$. *Đặt* $t=\sqrt[3]{\frac{1}{x^2}-1}\Rightarrow M=-\frac{3}{2}\int_{0}^{-\frac{\sqrt[3]{7}}{2}}t^3dt=-\frac{21\sqrt[3]{7}}{128}$

$N=\int_{1}^{2\sqrt{2}}\frac{2011}{x^3}dx=\int_{1}^{2\sqrt{2}}2011x^{-3}dx=\left[-\frac{2011}{2x^2}\right]\Bigg|_{1}^{2\sqrt{2}}=\frac{14077}{16}$

$\Rightarrow I=\frac{14077}{16}-\frac{21\sqrt[3]{7}}{128}$.

Câu 26. $I=\int_{0}^{1}\frac{dx}{(1+x^3).\sqrt[3]{1+x^3}}$

• *Đặt* $t=\sqrt[3]{1+x^3}\Rightarrow I=\int_{1}^{\sqrt[3]{2}}\frac{t^2}{t^4.(t^3-1)^{\frac{2}{3}}}dt=\int_{1}^{\sqrt[3]{2}}\frac{dt}{t^2.(t^3-1)^{\frac{2}{3}}}$

$$= \int_1^{\sqrt[3]{2}} \frac{dt}{t^2.\left[t^3\left(1-\frac{1}{t^3}\right)\right]^{\frac{2}{3}}} = \int_1^{\sqrt[3]{2}} \frac{dt}{t^4\left(1-\frac{1}{t^3}\right)^{\frac{2}{3}}} = \int_1^{\sqrt[3]{2}} \frac{\left(1-\frac{1}{t^3}\right)^{-\frac{2}{3}}}{t^4}dt$$

Đặt $u = 1-\frac{1}{t^3} \Rightarrow du = \frac{3dt}{t^4} \Rightarrow I = \int_0^{\frac{1}{2}} \frac{u^{-\frac{2}{3}}}{3}du = \frac{1}{3}\int_0^{\frac{1}{2}} u^{-\frac{2}{3}}du = \frac{1}{3}\left.\left(\frac{u^{\frac{1}{3}}}{\frac{1}{3}}\right)\right|_0^{\frac{1}{2}} = \left.u^{\frac{1}{3}}\right|_0^{\frac{1}{2}} = \frac{1}{\sqrt[3]{2}}$

Câu 27. $I = \int_{\sqrt{3}}^{2\sqrt{2}} \frac{x^4}{\left(x-\frac{1}{x}\right)\sqrt{x^2+1}}dx$

- *Đặt* $t = \sqrt{x^2+1}$

$\Rightarrow I = \int_2^3 \frac{(t^2-1)^2}{t^2-2}dt = \int_2^3 \frac{t^4-2t^2+1}{t^2-2}dt = \int_2^3 t^2dt + \int_2^3 \frac{1}{t^2-2}dt = \frac{19}{3} + \frac{\sqrt{2}}{4}\ln\left(\frac{4+\sqrt{2}}{4-\sqrt{2}}\right)$

Dạng 2: Đổi biến số dạng 2

Câu 28. $I = \int_0^1 \left(\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} - 2x\ln\left(1+x\right)\right)dx$

- *Tính* $H = \int_0^1 \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$. *Đặt* $\sqrt{x} = \cos t; t \in \left[0;\frac{\pi}{2}\right] \Rightarrow H = 2-\frac{\pi}{2}$
- *Tính* $K = \int_0^1 2x\ln(1+x)dx$. *Đặt* $\begin{cases} u = \ln(1+x) \\ dv = 2xdx \end{cases} \Rightarrow K = \frac{1}{2}$

Câu 29. $I = \int_{-2}^{2} (x^5+x^2)\sqrt{4-x^2}dx$

- $I = \int_{-2}^{2} (x^5+x^2)\sqrt{4-x^2}dx = \int_{-2}^{2} x^5\sqrt{4-x^2}dx + \int_{-2}^{2} x^2\sqrt{4-x^2}dx = A + B.$

\+ *Tính* $A = \int_{-2}^{2} x^5\sqrt{4-x^2}dx$. *Đặt* $t = -x$. *Tính được:* $A = 0.$

\+ *Tính* $B = \int_{-2}^{2} x^2\sqrt{4-x^2}dx$. *Đặt* $x = 2\sin t$. *Tính được:* $B = 2\pi$.

Vậy: $I = 2\pi$.

Câu 30. $I=\int_1^2 \frac{\left(3-\sqrt{4-x^2}\right)dx}{2x^4}$

• *Ta có:* $I=\int_1^2 \frac{3}{2x^4}dx-\int_1^2 \frac{\sqrt{4-x^2}}{2x^4}dx$.

+ *Tính* $I_1=\int_1^2 \frac{3}{2x^4}dx=\frac{3}{2}\int_1^2 x^{-4}dx=\frac{7}{16}$.

+ *Tính* $I_2=\int_1^2 \frac{\sqrt{4-x^2}}{2x^4}dx$. *Đặt* $x=2\sin t\Rightarrow dx=2\cos tdt$.

$\Rightarrow I_2=\frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^2 tdt}{\sin^4 t}=\frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t\left(\frac{1}{\sin^2 t}\right)dt=-\frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t.d(\cot t)=\frac{\sqrt{3}}{8}$

Vậy: $I=\frac{1}{16}\left(7-2\sqrt{3}\right)$.

Câu 31. $I=\int_0^1 \frac{x^2dx}{\sqrt{4-x^6}}$

• *Đặt* $t=x^3\Rightarrow dt=3x^2dx\ \Rightarrow I=\frac{1}{3}\int_0^1 \frac{dt}{\sqrt{4-t^2}}$.

Đặt $t=2\sin u,\ u\in\left[0;\frac{\pi}{2}\right]\Rightarrow dt=2\cos udu\Rightarrow I=\frac{1}{3}\int_0^{\frac{\pi}{6}} dt=\frac{\pi}{18}$.

Câu 32. $I=\int_0^2 \sqrt{\frac{2-x}{x+2}}dx$ • *Đặt* $x=2\cos t\Rightarrow dx=-2\sin tdt\ \Rightarrow I=4\int_0^{\frac{\pi}{2}} \sin^2\frac{t}{2}dt=\pi-2$.

Câu 33. $I=\int_0^1 \frac{x^2dx}{\sqrt{3+2x-x^2}}$

• *Ta có:* $I=\int_0^1 \frac{x^2dx}{\sqrt{2^2-(x-1)^2}}$. *Đặt* $x-1=2\cos t$.

$\Rightarrow I=-\int_{\frac{2\pi}{3}}^{\frac{\pi}{2}} \frac{(1+2\cos t)^2 2\sin t}{\sqrt{4-(2\cos t)^2}}dt=\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \left(3+4\cos t+2\cos 2t\right)dt=\frac{\pi}{2}+\frac{3\sqrt{3}}{2}-4$

Câu 34. $\int_0^{\frac{1}{2}} \sqrt{1-2x\sqrt{1-x^2}}\,dx$ • *Đặt* $x=\sin t\ \Rightarrow I=\int_0^{\frac{\pi}{6}} (\cos t-\sin t)\cos tdt=\frac{\pi}{12}+\frac{\sqrt{3}}{8}-\frac{1}{8}$

Dạng 3: Tích phân từng phần

Câu 35. $I = \int_{\sqrt{2}}^{3} \sqrt{x^2 - 1}dx$

• *Đặt* $\begin{cases} u = \sqrt{x^2 - 1} \\ dv = dx \end{cases} \Rightarrow \begin{cases} du = \dfrac{x}{\sqrt{x^2 - 1}}dx \\ v = x \end{cases}$

$$\Rightarrow I = x\sqrt{x^2 - 1}\Big|_{\sqrt{2}}^{3} - \int_{\sqrt{2}}^{3} x.\frac{x}{\sqrt{x^2 - 1}}dx = 5\sqrt{2} - \int_{\sqrt{2}}^{3}\left[\sqrt{x^2 - 1} + \frac{1}{\sqrt{x^2 - 1}}\right]dx$$

$$= 5\sqrt{2} - \int_{\sqrt{2}}^{3}\sqrt{x^2 - 1}dx - \int_{\sqrt{2}}^{3}\frac{dx}{\sqrt{x^2 - 1}} = 5\sqrt{2} - I - \ln\left|x + \sqrt{x^2 - 1}\right|\Big|_{\sqrt{2}}^{3}$$

$$\Rightarrow I = \frac{5\sqrt{2}}{2} - \ln\left(\sqrt{2} + 1\right) + \frac{1}{4}\ln 2$$

Chú ý: Không được dùng phép đổi biến $x = \dfrac{1}{\cos t}$ *vì* $\left[\sqrt{2};3\right] \notin \left[-1;1\right]$

TP3: TÍCH PHÂN HÀM SỐ LƯỢNG GIÁC

Dạng 1: Biến đổi lượng giác

Câu 1. $I=\int \frac{8\cos^2 x-\sin 2x-3}{\sin x-\cos x}dx$

• $I=\int \frac{(\sin x-\cos x)^2+4\cos 2x}{\sin x-\cos x}dx=\int\left[\left(\sin x-\cos x-4(\sin x+\cos x)\right)\right]dx$
$=3\cos x-5\sin x+C$.

Câu 2. $I=\int \frac{\cot x-\tan x-2\tan 2x}{\sin 4x}dx$

• *Ta có:* $I=\int \frac{2\cot 2x-2\tan 2x}{\sin 4x}dx=\int \frac{2\cot 4x}{\sin 4x}dx=2\int \frac{\cos 4x}{\sin^2 4x}dx=-\frac{1}{2\sin 4x}+C$

Câu 3. $I=\int \frac{\cos^2\left(x+\frac{\pi}{8}\right)}{\sin 2x+\cos 2x+\sqrt{2}}dx$

• *Ta có:* $I=\frac{1}{2\sqrt{2}}\int \frac{1+\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx$

$=\frac{1}{2\sqrt{2}}\left(\int \frac{\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx+\int \frac{dx}{\left[\sin\left(x+\frac{\pi}{8}\right)+\cos\left(x+\frac{\pi}{8}\right)\right]^2}\right)$

$=\frac{1}{2\sqrt{2}}\left(\int \frac{\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx+\frac{1}{2}\int \frac{dx}{\sin^2\left(x+\frac{3\pi}{8}\right)}\right)$

$=\frac{1}{4\sqrt{2}}\left(\ln\left|1+\sin\left(2x+\frac{\pi}{4}\right)\right|-\cot\left(x+\frac{3\pi}{8}\right)\right)+C$

Câu 4. $I=\int_{\frac{\pi}{3}}^{\pi} \frac{dx}{2+\sqrt{3}\sin x-\cos x}$

• $I=\frac{1}{2}\int_{\frac{\pi}{3}}^{\pi} \frac{dx}{1-\cos\left(x+\frac{\pi}{3}\right)}=I=\frac{1}{4}\int_{\frac{\pi}{3}}^{\pi} \frac{dx}{2\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right)}=\frac{1}{4\sqrt{3}}$.

Câu 5. $I=\int_0^{\frac{\pi}{6}} \frac{1}{2\sin x-\sqrt{3}}dx$

• *Ta có:* $I=\frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{1}{\sin x-\sin\frac{\pi}{3}}dx=\int_0^{\frac{\pi}{6}} \frac{\frac{1}{2}}{\sin x-\sin\frac{\pi}{3}}dx$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\frac{\pi}{3}}{\sin x - \sin\frac{\pi}{3}} dx = \int_0^{\frac{\pi}{6}} \frac{\cos\left(\left(\frac{x}{2}+\frac{\pi}{6}\right)-\left(\frac{x}{2}-\frac{\pi}{6}\right)\right)}{2\cos\left(\frac{x}{2}+\frac{\pi}{6}\right).\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)} dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)}{\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)} dx + \frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)} dx = \ln\left|\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)\right| \Big|_0^{\frac{\pi}{6}} - \ln\left|\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| \Big|_0^{\frac{\pi}{6}} = ...$$

Câu 6. $I = \int_0^{\frac{\pi}{2}} (\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x)dx$.

• *Ta có:* $(\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) = \frac{33}{64} + \frac{7}{16}\cos 4x + \frac{3}{64}\cos 8x \Rightarrow I = \frac{33}{128}\pi$.

Câu 7. $I = \int_0^{\frac{\pi}{2}} \cos 2x(\sin^4 x + \cos^4 x)dx$

• $I = \int_0^{\frac{\pi}{2}} \cos 2x\left(1 - \frac{1}{2}\sin^2 2x\right)dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}\left(1 - \frac{1}{2}\sin^2 2x\right)d(\sin 2x) = 0$

Câu 8. $I = \int_0^{\frac{\pi}{2}} (\cos^3 x - 1)\cos^2 x.dx$

• $A = \int_0^{\frac{\pi}{2}} \cos^5 x dx = \int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)^2 d(\sin x) = \frac{8}{15}$

$B = \int_0^{\frac{\pi}{2}} \cos^2 x.dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2x).dx = \frac{\pi}{4}$

Vậy $I = \frac{8}{15} - \frac{\pi}{4}$.

Câu 9. $\text{I} = \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx$

• $I = \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2x)\cos 2x dx = \frac{1}{4}\int_0^{\frac{\pi}{2}} (1 + 2\cos 2x + \cos 4x)dx$

$= \frac{1}{4}\left(x + \sin 2x + \frac{1}{4}\sin 4x\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{8}$

Câu 10. $I = \int_0^{\frac{\pi}{2}} \frac{4\sin^3 x}{1 + \cos x} dx$

- $\dfrac{4\sin^3 x}{1+\cos x} = \dfrac{4\sin^3 x(1-\cos x)}{\sin^2 x} = 4\sin x - 4\sin x\cos x = 4\sin x - 2\sin 2x$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} (4\sin x - 2\sin 2x)dx = 2$

Câu 11. $I = \int\limits_0^{2\pi} \sqrt{1+\sin x}dx$

- $I = \int\limits_0^{2\pi} \sqrt{\left(\sin\dfrac{x}{2} + \cos\dfrac{x}{2}\right)^2} dx = \int\limits_0^{2\pi} \left|\sin\dfrac{x}{2} + \cos\dfrac{x}{2}\right| dx = \sqrt{2}\int\limits_0^{2\pi} \left|\sin\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)\right| dx$

$= \sqrt{2}\left[\int\limits_0^{\frac{3\pi}{2}} \sin\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)dx - \int\limits_{\frac{3\pi}{2}}^{2\pi} \sin\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)dx\right] = 4\sqrt{2}$

Câu 12. $I = \int\limits_0^{\frac{\pi}{4}} \dfrac{dx}{\cos^6 x}$

- *Ta có:* $I = \int\limits_0^{\frac{\pi}{4}} (1 + 2\tan^2 x + \tan^4 x)d(\tan x) = \dfrac{28}{15}$.

Dạng 2: Đổi biến số dạng 1

Câu 13. $I = \int \dfrac{\sin 2x dx}{3 + 4\sin x - \cos 2x}$

- *Ta có:* $I = \int \dfrac{2\sin x\cos x}{2\sin^2 x + 4\sin x + 2}dx$. *Đặt* $t = \sin x \Rightarrow I = \ln|\sin x + 1| + \dfrac{1}{\sin x + 1} + C$

Câu 14. $I = \int \dfrac{dx}{\sin^3 x.\cos^5 x}$

- $I = \int \dfrac{dx}{\sin^3 x.\cos^3 x.\cos^2 x} = 8\int \dfrac{dx}{\sin^3 2x.\cos^2 x}$

Đặt $t = \tan x$. $I = \int\left(t^3 + 3t + \dfrac{3}{t} + t^{-3}\right)dt = \dfrac{1}{4}\tan^4 x + \dfrac{3}{2}\tan^2 x + 3\ln|\tan x| - \dfrac{1}{2\tan^2 x} + C$

Chú ý: $\sin 2x = \dfrac{2t}{1+t^2}$.

Câu 15. $I = \int \dfrac{dx}{\sin x.\cos^3 x}$

- $I = \int \dfrac{dx}{\sin x.\cos x.\cos^2 x} = 2\int \dfrac{dx}{\sin 2x.\cos^2 x}$. *Đặt* $t = \tan x \Rightarrow dt = \dfrac{dx}{\cos^2 x}$; $\sin 2x = \dfrac{2t}{1+t^2}$

$\Rightarrow I = 2\int \dfrac{dt}{\dfrac{2t}{1+t^2}} = \int \dfrac{t^2+1}{t}dt = \int (t + \dfrac{1}{t})dt = \dfrac{t^2}{2} + \ln|t| + C = \dfrac{\tan^2 x}{2} + \ln|\tan x| + C$

Câu 16. $I=\int \frac{\sqrt[2011]{\sin^{2011}x-\sin^{2009}x}}{\sin^5 x}\cot xdx$

• *Ta có:* $I=\int \frac{\sqrt[2011]{1-\frac{1}{\sin^2 x}}}{\sin^4 x}\cot xdx=\int \frac{\sqrt[2011]{-\cot^2 x}}{\sin^4 x}\cot xdx$

Đặt $t=\cot x\Rightarrow I=\int \text{t}^{\frac{2}{2011}}(1+t^2)tdt=\frac{2011}{4024}t^{\frac{4024}{2011}}+\frac{2011}{8046}t^{\frac{8046}{2011}}+C$

$=\frac{2011}{4024}\cot^{\frac{4024}{2011}}x+\frac{2011}{8046}\cot^{\frac{8046}{2011}}x+C$

Câu 17. $I=\int_0^{\frac{\pi}{2}}\frac{\sin 2x.\cos x}{1+\cos x}dx$

• *Ta có:* $I=2\int_0^{\frac{\pi}{2}}\frac{\sin x.\cos^2 x}{1+\cos x}dx$. *Đặt* $t=1+\cos x\Rightarrow I=2\int_1^2\frac{(t-1)^2}{t}dt=2\ln 2-1$

Câu 18. $I=\int_0^{\frac{\pi}{3}}\sin^2 x\tan xdx$

• *Ta có:* $I=\int_0^{\frac{\pi}{3}}\sin^2 x.\frac{\sin x}{\cos x}dx=\int_0^{\frac{\pi}{3}}\frac{(1-\cos^2 x)\sin x}{\cos x}dx$. *Đặt* $t=\cos x$

$\Rightarrow I=-\int_1^{\frac{1}{2}}\frac{1-u^2}{u}du=\ln 2-\frac{3}{8}$

Câu 19. $I=\int_{\frac{\pi}{2}}^{\pi}\sin^2 x(2-\sqrt{1+\cos 2x})dx$

• *Ta có:* $I=\int_{\frac{\pi}{2}}^{\pi}2\sin^2 xdx-\int_{\frac{\pi}{2}}^{\pi}\sin^2 x\sqrt{1+\cos 2x}dx=H+K$

$+\ H=\int_{\frac{\pi}{2}}^{\pi}2\sin^2 xdx=\int_{\frac{\pi}{2}}^{\pi}(1-\cos 2x)dx=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

$+\ K=\int_{\frac{\pi}{2}}^{\pi}\sin^2 x\sqrt{2\cos^2 x}=-\sqrt{2}\int_{\frac{\pi}{2}}^{\pi}\sin^2 x\cos xdx\ =-\sqrt{2}\int_{\frac{\pi}{2}}^{\pi}\sin^2 xd(\sin x)=\frac{\sqrt{2}}{3}$

$\Rightarrow I=\frac{\pi}{2}-\frac{\sqrt{2}}{3}$

Câu 20. $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x.\cos^4 x}$

• $I = 4.\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 2x.\cos^2 x}$. *Đặt* $t = \tan x \Rightarrow dt = \frac{dx}{\cos^2 x}$.

$I = \int_1^{\sqrt{3}} \frac{(1+t^2)^2 dt}{t^2} = \int_1^{\sqrt{3}} \left(\frac{1}{t^2} + 2 + t^2\right) dt = \left(-\frac{1}{t} + 2t + \frac{t^3}{3}\right)\Bigg|_1^{\sqrt{3}} = \frac{8\sqrt{3}-4}{3}$

Câu 21. $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{(2+\sin x)^2} dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{(2+\sin x)^2} dx = 2\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{(2+\sin x)^2} dx$. *Đặt* $t = 2 + \sin x$.

$\Rightarrow I = 2\int_2^3 \frac{t-2}{t^2} dt = 2\int_2^3 \left(\frac{1}{t} - \frac{2}{t^2}\right) dt = 2\left(\ln t + \frac{2}{t}\right)\Bigg|_2^3 = 2\ln\frac{3}{2} - \frac{2}{3}$

Câu 22. $I = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos 2x} dx$

• $I = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos 2x} dx = \int_0^{\frac{\pi}{6}} \frac{\sin x}{2\cos^2 x - 1} dx$. *Đặt* $t = \cos x \Rightarrow dt = -\sin x dx$

Đổi cận: $x = 0 \Rightarrow t = 1$; $x = \frac{\pi}{6} \Rightarrow t = \frac{\sqrt{3}}{2}$

Ta được $I = -\int_1^{\frac{\sqrt{3}}{2}} \frac{1}{2t^2 - 1} dt = \frac{1}{2\sqrt{2}} \ln\left|\frac{2t - \sqrt{2}}{2t + \sqrt{2}}\right|\Bigg|_{\frac{\sqrt{3}}{2}}^1 = \frac{1}{2\sqrt{2}} \ln\left|\frac{3 - 2\sqrt{2}}{5 - 2\sqrt{6}}\right|$

Câu 23. $I = \int_0^{\frac{\pi}{2}} e^{\sin^2 x}.\sin x.\cos^3 x.\, dx$ • *Đặt* $t = \sin^2 x \Rightarrow I = \frac{1}{2}\int_0^1 e^t (1-t) dt = \frac{1}{2}e - 1$.

Câu 24. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot \sqrt{\sin^2 x + \frac{1}{2}}\, dx$ • *Đặt* $t = \cos x$. $I = \frac{3}{16}(\pi + 2)$

Câu 25. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^6 x + \cos^6 x}} dx$

• $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{1-\frac{3}{4}\sin^2 2x}}dx$. Đặt $t = 1-\frac{3}{4}\sin^2 2x \Rightarrow \text{I} = \int_1^{\frac{1}{4}} \left(-\frac{2}{3}\frac{1}{\sqrt{t}}\right)dt = \frac{4}{3}\sqrt{t}\Big|_{\frac{1}{4}}^{1} = \frac{2}{3}$.

Câu 26. $I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\left(\sin x+\sqrt{3}\cos x\right)^3}dx$

• *Ta có:* $\sin x+\sqrt{3}\cos x = 2\cos\left(x-\frac{\pi}{6}\right)$;

$\sin x = \sin\left(\left(x-\frac{\pi}{6}\right)+\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\sin\left(x-\frac{\pi}{6}\right)+\frac{1}{2}\cos\left(x-\frac{\pi}{6}\right)$

$\Rightarrow I = \frac{\sqrt{3}}{16}\int_0^{\frac{\pi}{2}} \frac{\sin\left(x-\frac{\pi}{6}\right)dx}{\cos^3\left(x-\frac{\pi}{6}\right)} + \frac{1}{16}\int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2\left(x-\frac{\pi}{6}\right)} = \frac{\sqrt{3}}{6}$

Câu 27. $I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x\sqrt{1-\cos^2 x}}{\cos^2 x}dx$

• $I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x}\sqrt{1-\cos^2 x}.dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x}|\sin x|dx = \int_{-\frac{\pi}{3}}^{0} \frac{\sin x}{\cos^2 x}|\sin x|dx + \int_{-0}^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x}|\sin x|dx$

$= -\int_{-\frac{\pi}{3}}^{0} \frac{\sin^2 x}{\cos^2 x}dx + \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x}dx = \frac{7\pi}{12}-\sqrt{3}-1.$

Câu 28. $I = \int_0^{\frac{\pi}{6}} \frac{1}{\sin x+\sqrt{3}\cos x}dx$

• $I = \int_0^{\frac{\pi}{6}} \frac{1}{\sin x+\sqrt{3}\cos x}dx = \frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{1}{\sin\left(x+\frac{\pi}{3}\right)}dx = \frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{\sin\left(x+\frac{\pi}{3}\right)}{1-\cos^2\left(x+\frac{\pi}{3}\right)}dx$.

Đặt $t = \cos\left(x+\frac{\pi}{3}\right) \Rightarrow dt = -\sin\left(x+\frac{\pi}{3}\right)dx \Rightarrow I = \frac{1}{2}\int_0^{\frac{1}{2}} \frac{1}{1-t^2}dt = \frac{1}{4}\ln 3$

Câu 29. $I = \int_0^{\frac{\pi}{2}} \sqrt{1-\sqrt{3}\sin 2x+2\cos^2 x}dx$

• $I = \int_0^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx = I = \int_0^{\frac{\pi}{3}} \left|\sin x - \sqrt{3}\cos x\right| dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx = 3 - \sqrt{3}$

Câu 30. $I = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{(\sin x + \cos x)^3}$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt \Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos t dt}{(\sin t + \cos t)^3} = \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{(\sin x + \cos x)^3}$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{dx}{(\sin x + \cos x)^2} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{dx}{\sin^2(x + \frac{\pi}{4})} = -\frac{1}{2}\cot(x + \frac{\pi}{4})\Big|_0^{\frac{\pi}{4}} = 1 \Rightarrow I = \frac{1}{2}$

Câu 31. $I = \int_0^{\frac{\pi}{2}} \frac{7\sin x - 5\cos x}{(\sin x + \cos x)^3} dx$

• *Xét:* $I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{\left(\sin x + \cos x\right)^3}$; $\quad I_2 = \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{\left(\sin x + \cos x\right)^3}$.

Đặt $x = \frac{\pi}{2} - t$. *Ta chứng minh được* $I_1 = I_2$

Tính $I_1 + I_2 = \int_0^{\frac{\pi}{2}} \frac{dx}{\left(\sin x + \cos x\right)^2} = \int_0^{\frac{\pi}{2}} \frac{dx}{2\cos^2(x - \frac{\pi}{4})} = \frac{1}{2}\tan(x - \frac{\pi}{4})\Big|_0^{\frac{\pi}{2}} = 1$

$\Rightarrow I_1 = I_2 = \frac{1}{2} \Rightarrow I = 7I_1 - 5I_2 = 1.$

Câu 32. $I = \int_0^{\frac{\pi}{2}} \frac{3\sin x - 2\cos x}{(\sin x + \cos x)^3} dx$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt \Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{3\cos t - 2\sin t}{(\cos t + \sin t)^3} dt = \int_0^{\frac{\pi}{2}} \frac{3\cos x - 2\sin x}{(\cos x + \sin x)^3} dx$

$\Rightarrow 2I = I + I = \int_0^{\frac{\pi}{2}} \frac{3\sin x - 2\cos x}{(\sin x + \cos x)^3} dx + \int_0^{\frac{\pi}{2}} \frac{3\cos x - 2\sin x}{(\cos x + \sin x)^3} dx = \int_0^{\frac{\pi}{2}} \frac{1}{(\sin x + \cos x)^2} dx = 1 \Rightarrow I = \frac{1}{2}.$

Câu 33. $I = \int_0^{\pi} \frac{x\sin x}{1 + \cos^2 x} dx$

• *Đặt* $x = \pi - t \Rightarrow dx = -dt \Rightarrow I = \int_0^{\pi} \frac{(\pi - t)\sin t}{1 + \cos^2 t} dt = \pi\int_0^{\pi} \frac{\sin t}{1 + \cos^2 t} dt - I$

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin t}{1+\cos^2 t}dt = -\pi\int_0^{\pi} \frac{d(\cos t)}{1+\cos^2 t} = \pi\left(\frac{\pi}{4}+\frac{\pi}{4}\right) \Rightarrow I = \frac{\pi^2}{8}$$

Câu 34. $I = \int_0^{\frac{\pi}{2}} \frac{\cos^4 x \sin x}{\cos^3 x + \sin^3 x}dx$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt \Rightarrow I = -\int_{\frac{\pi}{2}}^{0} \frac{\sin^4 t \cos t}{\cos^3 t + \sin^3 t}dt = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x \cos x}{\cos^3 x + \sin^3 x}dx$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\cos^4 x \sin x + \sin^4 x \cos x}{\sin^3 x + \cos^3 x}dx = \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x(\sin^3 x + \cos^3 x)}{\sin^3 x + \cos^3 x}dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2x dx = \frac{1}{2}$$

$\Rightarrow I = \frac{1}{4}$.

Câu 35. $I = \int_0^{\frac{\pi}{2}} \left[\frac{1}{\cos^2(\sin x)} - \tan^2(\cos x)\right]dx$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left[\frac{1}{\cos^2(\cos t)} - \tan^2(\sin t)\right]dt \ = \int_0^{\frac{\pi}{2}} \left[\frac{1}{\cos^2(\cos x)} - \tan^2(\sin x)\right]dx$$

Do đó: $2I = \int_0^{\frac{\pi}{2}} \left[\frac{1}{\cos^2(\sin x)} + \frac{1}{\cos^2(\cos x)} - \tan^2(\cos x) - \tan^2(\sin x)\right]dx \ = 2\int_0^{\frac{\pi}{2}} dt = \pi$

$\Rightarrow I = \frac{\pi}{2}$.

Câu 36. $I = \int_0^{\frac{\pi}{4}} \frac{\cos x - \sin x}{\sqrt{3 - \sin 2x}}dx$

• Đặt $u = \sin x + \cos x \ \Rightarrow I = \int_1^{\sqrt{2}} \frac{du}{\sqrt{4-u^2}}$. Đặt $u = 2\sin t \ \Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{2\cos t dt}{\sqrt{4-4\sin^2 t}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} dt = \frac{\pi}{12}$.

Câu 37. $I = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x\sqrt{3+\sin^2 x}}dx$

• *Đặt* $t = \sqrt{3+\sin^2 x} = \sqrt{4-\cos^2 x}$. *Ta có:* $\cos^2 x = 4 - t^2$ *và* $dt = \frac{\sin x \cos x}{\sqrt{3+\sin^2 x}}dx$.

$$I = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x\sqrt{3+\sin^2 x}}.dx = \int_0^{\frac{\pi}{3}} \frac{\sin x.\cos x}{\cos^2 x\sqrt{3+\sin^2 x}}dx = \int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}} \frac{dt}{4-t^2} = \frac{1}{4}\int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}} \left(\frac{1}{t+2} - \frac{1}{t-2}\right)dt$$

$= \frac{1}{4}\ln\left|\frac{t+2}{t-2}\right|\Bigg|_{\sqrt{3}}^{\frac{\sqrt{15}}{2}} = \frac{1}{4}\left(\ln\left|\frac{\sqrt{15}+4}{\sqrt{15}-4}\right| - \ln\left|\frac{\sqrt{3}+2}{\sqrt{3}-2}\right|\right) = \frac{1}{2}\left(\ln\left(\sqrt{15}+4\right) - \ln\left(\sqrt{3}+2\right)\right).$

Câu 38. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x+\sin x)\sin x}{\sin^3 x + \sin^2 x} dx$

• $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x}$.

+ *Tính* $I_1 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx$. *Đặt* $\begin{cases} u = x \\ dv = \dfrac{dx}{\sin^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\cot x \end{cases} \Rightarrow I_1 = \frac{\pi}{\sqrt{3}}$

+ *Tính* $I_2 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} = 4-2\sqrt{3}$

Vậy: $I = \frac{\pi}{\sqrt{3}} + 4 - 2\sqrt{3}$.

Câu 39. $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{\cos^2 x + 4\sin^2 x}} dx$

• $I = \int_0^{\frac{\pi}{2}} \frac{2\sin x\cos x}{\sqrt{3\sin^2 x + 1}} dx$. Đặt $u = \sqrt{3\sin^2 x + 1} \Rightarrow I = \int_1^2 \frac{\frac{2}{3}udu}{u} = \frac{2}{3}\int_1^2 du = \frac{2}{3}$

Câu 40. $I = \int_0^{\frac{\pi}{6}} \frac{\tan\left(x - \frac{\pi}{4}\right)}{\cos 2x} dx$

• $I = \int_0^{\frac{\pi}{6}} \frac{\tan\left(x-\frac{\pi}{4}\right)}{\cos 2x} dx = -\int_0^{\frac{\pi}{6}} \frac{\tan^2 x + 1}{(\tan x + 1)^2} dx$. *Đặt* $t = \tan x \Rightarrow dt = \frac{1}{\cos^2 x} dx = (\tan^2 x + 1)dx$

$\Rightarrow I = -\int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{(t+1)^2} = \frac{1}{t+1}\Bigg|_0^{\frac{1}{\sqrt{3}}} = \frac{1-\sqrt{3}}{2}$.

Câu 41. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x.\sin\left(x + \frac{\pi}{4}\right)} dx$

• $I = \sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin^2 x(1+\cot x)} dx$. *Đặt* $1 + \cot x = t \Rightarrow \frac{1}{\sin^2 x} dx = -dt$

$\Rightarrow I = \sqrt{2}\int_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} \frac{t-1}{t} dt = \sqrt{2}\left(t - \ln t\right)\Bigg|_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} = \sqrt{2}\left(\frac{2}{\sqrt{3}} - \ln\sqrt{3}\right)$

Câu 42. $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x.\cos^4 x}$

• *Ta có:* $I = 4.\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 2x.\cos^2 x}$. *Đặt* $t = \tan x \Rightarrow dx = \frac{dt}{1+t^2}$

$\Rightarrow I = \int_1^{\sqrt{3}} \frac{(1+t^2)^2 dt}{t^2} = \int_1^{\sqrt{3}} (\frac{1}{t^2} + 2 + t^2)dt = (-\frac{1}{t} + 2t + \frac{t^3}{3})\Big|_1^{\sqrt{3}} = \frac{8\sqrt{3}-4}{3}$

Câu 43. $I = \int_0^{\frac{\pi}{4}} \frac{\sin x}{5\sin x.\cos^2 x + 2\cos x} dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{4}} \frac{\tan x}{5\tan x + 2(1+\tan^2 x)}.\frac{1}{\cos^2 x} dx$. *Đặt* $t = \tan x$,

$\Rightarrow I = \int_0^1 \frac{t}{2t^2 + 5t + 2} dt = \frac{1}{3}\int_0^1 \left(\frac{2}{t+2} - \frac{1}{2t+1}\right) dt = \frac{1}{2}\ln 3 - \frac{2}{3}\ln 2$

Câu 44. $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^2 x dx}{\cos^4 x(\tan^2 x - 2\tan x + 5)}$

• *Đặt* $t = \tan x \Rightarrow dx = \frac{dt}{1+t^2}$ $\Rightarrow I = \int_{-1}^{1} \frac{t^2 dt}{t^2 - 2t + 5} = 2 + \ln\frac{2}{3} - 3\int_{-1}^{1} \frac{dt}{t^2 - 2t + 5}$

Tính $I_1 = \int_{-1}^{1} \frac{dt}{t^2 - 2t + 5}$. *Đặt* $\frac{t-1}{2} = \tan u \Rightarrow I_1 = \frac{1}{2}\int_{-\frac{\pi}{4}}^{0} du = \frac{\pi}{8}$. *Vậy* $I = 2 + \ln\frac{2}{3} - \frac{3\pi}{8}$.

Câu 45. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin 3x} dx$.

• $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{3\sin x - 4\sin^3 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin x}{4\cos^2 x - 1} dx$

Đặt $t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow$ $I = -\int_{\frac{\sqrt{3}}{2}}^{0} \frac{dt}{4t^2 - 1} = \frac{1}{4}\int_0^{\frac{\sqrt{3}}{2}} \frac{dt}{t^2 - \frac{1}{4}} = \frac{1}{4}\ln(2-\sqrt{3})$

Câu 46. $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}} dx$

• *Ta có:* $\sqrt{1+\sin 2x} = |\sin x+\cos x| = \sin x+\cos x$ *(vì* $x \in \left[\frac{\pi}{4};\frac{\pi}{2}\right]$*)*

$\Rightarrow I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x-\cos x}{\sin x+\cos x}dx$. *Đặt* $t = \sin x+\cos x \Rightarrow dt = (\cos x-\sin x)dx$

$\Rightarrow I = \int_1^{\sqrt{2}} \frac{1}{t}dt = \ln|t|\Big|_1^{\sqrt{2}} = \frac{1}{2}\ln 2$

Câu 47. $I = 2\int_1^2 \sqrt[6]{1-\cos^3 x}.\sin x.\cos^5 x dx$

• *Đặt* $t = \sqrt[6]{1-\cos^3 x} \Leftrightarrow t^6 = 1-\cos^3 x \Rightarrow 6t^5dt = 3\cos^2 x\sin x dx \Rightarrow dx = \frac{2t^5dt}{\cos^2 x\sin x}$

$\Rightarrow I = 2\int_0^1 t^6(1-t^6)dt = 2\left(\frac{t^7}{7}-\frac{t^{13}}{13}\right)\Bigg|_0^1 = \frac{12}{91}$

Câu 48. $I = \int_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos x\sqrt{1+\cos^2 x}}$

• *Ta có:* $I = \int_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos^2 x\sqrt{\tan^2 x+2}}$. *Đặt* $t = \sqrt{2+\tan^2 x} \Rightarrow t^2 = 2+\tan^2 x \Rightarrow tdt = \frac{\tan x}{\cos^2 x}dx$

$\Rightarrow I = \int_{\sqrt{2}}^{\sqrt{3}} \frac{tdt}{t} = \int_{\sqrt{2}}^{\sqrt{3}} dt = \sqrt{3}-\sqrt{2}$

Câu 49. $I = \int_0^{\frac{\pi}{2}} \frac{\cos 2x}{(\cos x-\sin x+3)^3}dx$ • *Đặt* $t = \cos x-\sin x+3 \Rightarrow I = \int_2^4 \frac{t-3}{t^3}dt = -\frac{1}{32}$.

Câu 50. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\cos^2 x.\sqrt{\tan^4 x+1}}dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^4 x+\cos^4 x}}dx$. *Đặt* $t = \sqrt{\sin^4 x+\cos^4 x}$ $\Rightarrow I = -2\int_1^{\frac{\sqrt{2}}{2}} dt = 2-\sqrt{2}$.

Câu 51. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{1+\cos^2 x}dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{4}} \frac{2\sin 2x(2\cos^2 x-1)}{1+\cos^2 x}dx$. *Đặt* $t = \cos^2 x \Rightarrow I = -\int_1^{\frac{1}{2}} \frac{2(2t-1)}{t+1}dt = 2-6\ln\frac{1}{3}$.

Câu 52. $I = \int_0^{\frac{\pi}{6}} \frac{\tan(x-\frac{\pi}{4})}{\cos 2x}dx$

• *Ta có:* $I=-\int_0^{\frac{\pi}{6}}\frac{\tan^2x+1}{(\tan x+1)^2}dx$. *Đặt* $t=\tan x \Rightarrow I=-\int_0^{\frac{1}{\sqrt{3}}}\frac{dt}{(t+1)^2}=\frac{1-\sqrt{3}}{2}$.

Câu 53. $I=\int_0^{\frac{\pi}{6}}\frac{\tan^3x}{\cos 2x}dx$

• *Ta có:* $I=\int_0^{\frac{\pi}{6}}\frac{\tan^3x}{\cos^2x-\sin^2x}dx=\int_0^{\frac{\pi}{6}}\frac{\tan^3x}{\cos^2x(1-\tan^2x)}dx$.

Đặt $t=\tan x \Rightarrow I=\int_0^{\frac{\sqrt{3}}{3}}\frac{t^3}{1-t^2}dt=-\frac{1}{6}-\frac{1}{2}\ln\frac{2}{3}$.

Câu 54. $I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sqrt{7+\cos 2x}}dx$ • $I=\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}}\frac{\cos x\,dx}{\sqrt{2^2-\sin^2x}}=\frac{\pi}{6\sqrt{2}}$

Câu 55. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{dx}{\sqrt[4]{\sin^3x.\cos^5x}}$

• *Ta có:* $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sqrt[4]{\frac{\sin^3x}{\cos^3x}.\cos^8x}}dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sqrt[4]{\tan^3x}}.\frac{1}{\cos^2x}dx$.

Đặt $t=\tan x \Rightarrow I=\int_1^{\sqrt{3}}t^{-\frac{3}{4}}dt=4\left(\sqrt[8]{3}-1\right)$

Câu 56. $I=\int_0^{\pi}x(\frac{\cos^3x+\cos x+\sin x}{1+\cos^2x})dx$

• *Ta có:* $I=\int_0^{\pi}x\left(\frac{\cos x(1+\cos^2x)+\sin x}{1+\cos^2x}\right)dx=\int_0^{\pi}x.\cos x.dx+\int_0^{\pi}\frac{x.\sin x}{1+\cos^2x}dx=J+K$

+ *Tính* $J=\int_0^{\pi}x.\cos x.dx$. *Đặt* $\begin{cases}u=x\\dv=\cos xdx\end{cases}\Rightarrow\begin{cases}du=dx\\v=\sin x\end{cases}\Rightarrow J=-2$

+ *Tính* $K=\int_0^{\pi}\frac{x.\sin x}{1+\cos^2x}dx$. *Đặt* $x=\pi-t\Rightarrow dx=-dt$

$\Rightarrow K=\int_0^{\pi}\frac{(\pi-t).\sin(\pi-t)}{1+\cos^2(\pi-t)}dt=\int_0^{\pi}\frac{(\pi-t).\sin t}{1+\cos^2t}dt=\int_0^{\pi}\frac{(\pi-x).\sin x}{1+\cos^2x}dx$

$\Rightarrow 2K=\int_0^{\pi}\frac{(x+\pi-x).\sin x}{1+\cos^2x}dx=\pi\int_0^{\pi}\frac{\sin x.dx}{1+\cos^2x}\Rightarrow K=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x.dx}{1+\cos^2x}$

Đặt $t=\cos x \Rightarrow K=\frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2}$, *đặt* $t=\tan u \Rightarrow dt=(1+\tan^2 u)du$

$$\Rightarrow K=\frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan^2 u)du}{1+\tan^2 u}=\frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}du=\frac{\pi}{2}.u\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}=\frac{\pi^2}{4}$$

Vậy $I=\frac{\pi^2}{4}-2$

Câu 57. $\mathrm{I}=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos x}{\sin x\sqrt{3+\cos^2 x}}dx$

• *Ta có:* $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^2 x\sqrt{3+\cos^2 x}}dx$. *Đặt* $t=\sqrt{3+\cos^2 x}$

$$\Rightarrow I=\int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}}\frac{dt}{4-t^2}=\frac{1}{2}\left(\ln(\sqrt{15}+4)-\ln(\sqrt{3}+2)\right)$$

Dạng 3: Đổi biến số dạng 2

Câu 58. $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin x\cdot\sqrt{\sin^2 x+\frac{1}{2}}.dx$

• *Đặt* $\cos x=\sqrt{\frac{3}{2}}\sin t,\ \left(0\le t\le\frac{\pi}{2}\right)\Rightarrow I=\frac{3}{2}\int_{0}^{\frac{\pi}{4}}\cos^2 t\,dt=\frac{3}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right).$

Câu 59. $I=\int_{0}^{\frac{\pi}{2}}\frac{3\sin x+4\cos x}{3\sin^2 x+4\cos^2 x}dx$

• $I=\int_{0}^{\frac{\pi}{2}}\frac{3\sin x+4\cos x}{3+\cos^2 x}dx=\int_{0}^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx+\int_{0}^{\frac{\pi}{2}}\frac{4\cos x}{3+\cos^2 x}dx=\int_{0}^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx+\int_{0}^{\frac{\pi}{2}}\frac{4\cos x}{4-\sin^2 x}dx$

+ *Tính* $I_1=\int_{0}^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx$. *Đặt* $t=\cos x\Rightarrow dt=-\sin x dx \Rightarrow I_1=\int_{0}^{1}\frac{3dt}{3+t^2}$

Đặt $t=\sqrt{3}\tan u\Rightarrow dt=\sqrt{3}(1+\tan^2 u)du \Rightarrow I_1=\int_{0}^{\frac{\pi}{6}}\frac{3\sqrt{3}(1+\tan^2 u)du}{3(1+\tan^2 u)}=\frac{\pi\sqrt{3}}{6}$

+ *Tính* $I_2=\int_{0}^{\frac{\pi}{2}}\frac{4\cos x}{4-\sin^2 x}dx$. *Đặt* $t_1=\sin x\Rightarrow dt_1=\cos x dx$ $I_2=\int_{0}^{1}\frac{4dt_1}{4-t_1^{\,2}}dt_1=\ln 3$

Vậy: $I = \dfrac{\pi\sqrt{3}}{6} + \ln 3$

Câu 60. $I = \displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos x\sqrt{1+\cos^2 x}}dx$

• *Ta có:* $I = \displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos^2 x\sqrt{\frac{1}{\cos^2 x}+1}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos^2 x\sqrt{\tan^2 x+2}}dx$

Đặt $u = \tan x \Rightarrow du = \dfrac{1}{\cos^2 x}dx \Rightarrow I = \displaystyle\int_{\frac{1}{\sqrt{3}}}^{1} \frac{u}{\sqrt{u^2+2}}dx$. *Đặt* $t = \sqrt{u^2+2} \Rightarrow dt = \dfrac{u}{\sqrt{u^2+2}}du$.

$\Rightarrow I = \displaystyle\int_{\sqrt{\frac{7}{3}}}^{\sqrt{3}} dt = t\Big|_{\sqrt{\frac{7}{3}}}^{\sqrt{3}} = \sqrt{3} - \frac{\sqrt{7}}{\sqrt{3}} = \frac{3-\sqrt{7}}{\sqrt{3}}$.

Câu 61. $I = \displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin\left(x+\frac{\pi}{4}\right)}{2\sin x\cos x - 3}dx$

• *Ta có:* $I = -\dfrac{1}{\sqrt{2}}\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\left(\sin x - \cos x\right)^2 + 2}dx$. *Đặt* $t = \sin x - \cos x \Rightarrow I = -\dfrac{1}{\sqrt{2}}\displaystyle\int_0^1 \frac{1}{t^2+2}dt$

Đặt $t = \sqrt{2}\tan u \Rightarrow I = -\dfrac{1}{\sqrt{2}}\displaystyle\int_0^{\arctan\frac{1}{\sqrt{2}}} \frac{\sqrt{2}(1+\tan^2 u)}{2\tan^2 u + 2}du = -\frac{1}{2}\arctan\frac{1}{\sqrt{2}}$

Dạng 4: Tích phân từng phần

Câu 62. $I=\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}}\frac{x\sin x}{\cos^2 x}dx$.

• *Sử dụng công thức tích phân từng phần ta có:*

$$I=\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}xd\left(\frac{1}{\cos x}\right)=\frac{x}{\cos x}\Bigg|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}-\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{dx}{\cos x}=\frac{4\pi}{3}-J,\ \text{với}\ J=\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{dx}{\cos x}$$

Để tính J ta đặt $t=\sin x$. *Khi đó* $J=\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{dx}{\cos x}=\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}\frac{dt}{1-t^2}=-\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\Bigg|_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}=-\ln\frac{2-\sqrt{3}}{2+\sqrt{3}}$

Vậy $I=\frac{4\pi}{3}-\ln\frac{2-\sqrt{3}}{2+\sqrt{3}}$.

Câu 63. $I=\int_0^{\frac{\pi}{2}}\left(\frac{1+\sin x}{1+\cos x}\right).e^x dx$

• *Ta có:* $\frac{1+\sin x}{1+\cos x}=\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}=\frac{1}{2\cos^2\frac{x}{2}}+\tan\frac{x}{2}$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{e^x dx}{2\cos^2\frac{x}{2}}+\int_0^{\frac{\pi}{2}}e^x\tan\frac{x}{2}dx = e^{\frac{\pi}{2}}$$

Câu 64. $I=\int_0^{\frac{\pi}{4}}\frac{x\cos 2x}{(1+\sin 2x)^2}dx$

• *Đặt* $\begin{cases}u=x\\ dv=\dfrac{\cos 2x}{(1+\sin 2x)^2}dx\end{cases}\Rightarrow\begin{cases}du=dx\\ v=-\dfrac{1}{1+\sin 2x}\end{cases}$

$$\Rightarrow I=x.\left(-\frac{1}{2}.\frac{1}{1+\sin 2x}\right)\Bigg|_0^{\frac{\pi}{4}}+\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{1}{1+\sin 2x}dx=-\frac{\pi}{16}+\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{1}{\sqrt{2}}.\frac{1}{\cos^2\left(x-\frac{\pi}{4}\right)}dx$$

$$=-\frac{\pi}{16}+\frac{1}{2}.\frac{1}{\sqrt{2}}\tan\left(x-\frac{\pi}{4}\right)\Bigg|_0^{\frac{\pi}{4}}=-\frac{\pi}{16}+\frac{1}{2}.\frac{\sqrt{2}}{2}(0+1)=\frac{\sqrt{2}}{4}-\frac{\pi}{16}$$

TP4: TÍCH PHÂN HÀM SỐ MŨ - LOGARIT

Dạng 1: Đổi biến số

Câu 1. $I = \int \frac{e^{2x}}{1+\sqrt{e^x}} dx$

- *Đặt* $t = \sqrt{e^x} \Rightarrow e^x = t^2 \Rightarrow e^x dx = 2tdt$.

$\Rightarrow I = 2\int \frac{t^3}{1+t} dt = \frac{2}{3}t^3 - t^2 + 2t - 2\ln|t+1| + C = \frac{2}{3}e^x\sqrt{e^x} - e^x + 2\sqrt{e^x} - 2\ln\left|\sqrt{e^x}+1\right| + C$

Câu 2. $I = \int \frac{(x^2+x)e^x}{x+e^{-x}} dx$

- $I = \int \frac{(x^2+x)e^x}{x+e^{-x}} dx = \int \frac{xe^x.(x+1)e^x}{xe^x+1} dx$. *Đặt* $t = x.e^x + 1 \Rightarrow I = xe^x + 1 - \ln|xe^x+1| + C$.

Câu 3. $I = \int \frac{dx}{\sqrt{e^{2x}+9}}$

- *Đặt* $t = \sqrt{e^{2x}+9} \Rightarrow I = \int \frac{dt}{t^2-9} = \frac{1}{6}\ln\left|\frac{t-3}{t+3}\right| + C = \frac{1}{6}\ln\left|\frac{\sqrt{e^{2x}+9}-3}{\sqrt{e^{2x}+9}+3}\right| + C$

Câu 4. $I = \int \frac{\ln(1+x^2)^x + 2011x}{\ln\left[(ex^2+e)^{x^2+1}\right]} dx$

- *Ta có:* $I = \int \frac{x\left[\ln(x^2+1)+2011\right]}{(x^2+1)\left[\ln(x^2+1)+1\right]} dx$. *Đặt* $t = \ln(x^2+1)+1$

$\Rightarrow I = \frac{1}{2}\int \frac{t+2010}{t} dt = \frac{1}{2}t + 1005\ln|t| + C = \frac{1}{2}\ln(x^2+1) + \frac{1}{2} + 1005\ln(\ln(x^2+1)+1) + C$

Câu 5. $J = \int_1^e \frac{xe^x+1}{x(e^x+\ln x)} dx$

- $J = \int_1^e \frac{d(e^x+\ln x)}{e^x+\ln x} = \ln\left|e^x+\ln x\right|\Big|_1^e = \ln\frac{e^e+1}{e}$

Câu 6. $I = \int_0^{\ln 2} \frac{2e^{3x}+e^{2x}-1}{e^{3x}+e^{2x}-e^x+1} dx$

- $I = \int_0^{\ln 2} \frac{3e^{3x}+2e^{2x}-e^x-(e^{3x}+e^{2x}-e^x+1)}{e^{3x}+e^{2x}-e^x+1} dx = \int_0^{\ln 2} \left(\frac{3e^{3x}+2e^{2x}-e^x}{e^{3x}+e^{2x}-e^x+1} - 1\right) dx$

$= \ln(e^{3x}+e^{2x}-e^x+1)\Big|_0^{\ln 2} - x\Big|_0^{\ln 2} = \ln 11 - \ln 4 = \ln\frac{14}{4}$

Câu 7. $I = \int_0^{3\ln 2} \frac{dx}{\left(\sqrt[3]{e^x}+2\right)^2}$

- $I = \int_0^{3\ln 2} \frac{e^{\frac{x}{3}} dx}{e^{\frac{x}{3}}\left(\sqrt[3]{e^x}+2\right)^2}$. *Đặt* $t = e^{\frac{x}{3}} \Rightarrow dt = \frac{1}{3}e^{\frac{x}{3}} dx \Rightarrow I = \frac{3}{4}\left(\ln\frac{3}{2} - \frac{1}{6}\right)$

Câu 8. $I = \int_0^{\ln 2} \sqrt[3]{e^x - 1}\, dx$

• *Đặt* $\sqrt[3]{e^x - 1} = t \Rightarrow dx = \frac{3t^2dt}{t^3+1} \Rightarrow I = 3\int_0^1 \left(1 - \frac{1}{t^3+1}\right)dt = 3 - 3\int_0^1 \frac{dt}{t^3+1}$.

Tính $I_1 = 3\int_0^1 \frac{dt}{t^3+1} = \int_0^1 \left(\frac{1}{t+1} + \frac{2-t}{t^2-t+1}\right)dt = \frac{\pi}{\sqrt{3}} + \ln 2$

Vậy: $I = 3 - \ln 2 - \frac{\pi}{\sqrt{3}}$

Câu 9. $I = \int_{3\ln 2}^{\ln 15} \frac{\left(e^{2x} - 24e^x\right)dx}{e^x\sqrt{e^x+1} + 5e^x - 3\sqrt{e^x+1} - 15}$

• *Đặt* $t = \sqrt{e^x+1} \Rightarrow t^2 - 1 = e^x \Rightarrow e^x dx = 2tdt$.

$I = \int_3^4 \frac{(2t^2 - 10t)dt}{t^2-4} = \int_3^4 \left(2 - \frac{3}{t-2} - \frac{7}{t+2}\right)dt = \left(2t - 3\ln|t-2| - 7\ln|t+2|\right)\Big|_3^4$

$= 2 - 3\ln 2 - 7\ln 6 + 7\ln 5$

Câu 10. $I = \int_{\ln 2}^{\ln 3} \frac{e^{2x}dx}{e^x - 1 + \sqrt{e^x - 2}}$

• *Đặt* $t = \sqrt{e^x - 2} \Rightarrow e^{2x}dx = 2tdt$

$\Rightarrow I = 2\int_0^1 \frac{(t^2+2)tdt}{t^2+t+1} = 2\int_0^1 \left(t - 1 + \frac{2t+1}{t^2+t+1}\right)dt = 2\int_0^1 (t-1)dt + 2\int_0^1 \frac{d(t^2+t+1)}{t^2+t+1}$

$= (t^2 - 2t)\Big|_0^1 + 2\ln(t^2+t+1)\Big|_0^1 = 2\ln 3 - 1$.

Câu 11. $I = \int_0^{\ln 3} \frac{2e^{3x} - e^{2x}}{e^x\sqrt{4e^x - 3} + 1}dx$

• *Đặt* $t = \sqrt{4e^{3x} - 3e^{2x}} \Rightarrow t^2 = 4e^{3x} - 3e^{2x} \Rightarrow 2tdt = (12e^{3x} - 6e^{2x})dx \Rightarrow (2e^{3x} - e^{2x})dx = \frac{tdt}{3}$

$\Rightarrow I = \frac{1}{3}\int_1^9 \frac{tdt}{t+1} = \frac{1}{3}\int_1^9 (1 - \frac{1}{t+1})dt = \frac{1}{3}(t - \ln|t+1|)\Big|_1^9 = \frac{8 - \ln 5}{3}$.

Câu 12. $I = \int_{\ln\frac{8}{3}}^{\ln\frac{16}{3}} \sqrt{3e^x - 4}dx$

• *Đặt:* $t = \sqrt{3e^x - 4} \Rightarrow e^x = \frac{t^2+4}{3} \Rightarrow dx = \frac{2tdt}{t^2+4}$

$\Rightarrow I = \int_2^{2\sqrt{3}} \frac{2t^2}{t^2+4}dt = 2\int_2^{2\sqrt{3}} dt - 8\int_2^{2\sqrt{3}} \frac{dt}{t^2+4} = 4\left(\sqrt{3} - 1\right) - 8I_1$, *với* $I_1 = \int_2^{2\sqrt{3}} \frac{dt}{t^2+4}$

Tính $I_1 = \int_2^{2\sqrt{3}} \frac{dt}{t^2+4}$. *Đặt:* $t = 2\tan u,\ u \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right) \Rightarrow dt = 2(1 + \tan^2 u)du$

$\Rightarrow I_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{2} du = \frac{1}{2}\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{24}$. *Vậy:* $I = 4(\sqrt{3} - 1) - \frac{\pi}{3}$

Câu 13. $I = \int_0^{\ln 3} \frac{e^x}{\sqrt{(e^x + 1)^3}} dx$

• *Đặt* $t = \sqrt{e^x + 1} \Leftrightarrow t^2 = e^x + 1 \Leftrightarrow 2tdt = e^x dx \Rightarrow dx = \frac{2tdt}{e^x}$ $\Rightarrow I = 2\int_{\sqrt{2}}^{2} \frac{tdt}{t^3} = \sqrt{2} - 1$

Câu 14. $I = \int_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}} dx$

• *Đặt* $t = \sqrt{e^x - 1} \Leftrightarrow t^2 = e^x - 1 \Rightarrow dx = \frac{2tdt}{e^x} \Rightarrow I = 2\int_1^2 (t^2 + 1)d = 2\left(\frac{t^3}{3} + t\right)\Big|_1^2 = \frac{20}{3}$

Câu 15. $I = \int_0^{\ln 2} \sqrt{e^x - 1} dx$

• *Đặt* $t = \sqrt{e^x - 1} \Rightarrow t^2 = e^x - 1 \Rightarrow 2tdt = e^x dx \Rightarrow dx = \frac{2td}{e^x} = \frac{2td}{t^2 + 1}$

$\Rightarrow I = \int_0^1 \frac{2t^2}{t^2 + 1} dt = 2\int_0^1 \left(1 - \frac{1}{t^2 + 1}\right) dt = \frac{4 - \pi}{2}$

Câu 16. $I = \int_1^2 \frac{2^x - 2^{-x}}{4^x + 4^{-x} - 2} dx$

• *Đặt* $t = 2^x + 2^{-x} \Rightarrow 4^x + 4^{-x} - 2 = (2^x + 2^{-x})^2 - 4 \Rightarrow I = \frac{1}{4\ln 2}\ln\frac{81}{25}$

Câu 17. $I = \int_0^1 \frac{6^x dx}{9^x + 3.6^x + 2.4^x}$

• *Ta có:* $I = \int_0^1 \frac{\left(\frac{3}{2}\right)^x dx}{\left(\frac{3}{2}\right)^{2x} + 3\left(\frac{3}{2}\right)^x + 2}$. *Đặt* $t = \left(\frac{3}{2}\right)^x$. $I = \frac{1}{\ln 3 - \ln 2}\int_1^{\frac{3}{2}} \frac{dt}{t^2 + 3t + 2} = \frac{\ln 15 - \ln 14}{\ln 3 - \ln 2}$

Câu 18. $I = \int_1^e \left(\frac{\ln x}{x\sqrt{1 + \ln x}} + 3x^2 \ln x\right) dx$

• $I = \int_1^e \frac{\ln x}{x\sqrt{1 + \ln x}} dx + 3\int_1^e x^2 \ln x dx = \frac{2(2 - \sqrt{2})}{3} + \frac{2e^3 + 1}{3} = \frac{5 - 2\sqrt{2} + 2e^3}{3}$

Câu 19. $I = \int_1^e \frac{\ln x\sqrt[3]{2 + \ln^2 x}}{x} dx$

• *Đặt* $t = 2 + \ln^2 x \Rightarrow dt = \dfrac{2\ln x}{x}dx \Rightarrow I = \dfrac{1}{2}\int_2^3 \sqrt[3]{t}dt = \dfrac{3}{8}\left(\sqrt[3]{3^4} - \sqrt[3]{2^4}\right)$

Câu 20. $I = \displaystyle\int_e^{e^2} \dfrac{dx}{x\ln x.\ln ex}$

• $I = \displaystyle\int_e^{e^2} \dfrac{dx}{x\ln x(1+\ln x)} = \int_e^{e^2} \dfrac{d(\ln x)}{\ln x(1+\ln x)} = \int_e^{e^2}\left(\dfrac{1}{\ln x} - \dfrac{1}{1+\ln x}\right)d(\ln x) = 2\ln 2 - \ln 3$

Câu 21. $I = \displaystyle\int_{\ln 4}^{\ln 6} \dfrac{e^{2x}}{e^x + 6e^{-x} - 5}dx$ • *Đặt* $t = e^x$. $I = 2 + 9\ln 3 - 4\ln 2$

Câu 22. $I = \displaystyle\int_1^e \dfrac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}}dx$

• $I = \displaystyle\int_1^e \dfrac{\log_2^3 x}{x\sqrt{1+3\ln^2 x}}dx = \int_1^e \dfrac{\left(\dfrac{\ln x}{\ln 2}\right)^3}{x\sqrt{1+3\ln^2 x}}dx = \dfrac{1}{\ln^3 2}\int_1^e \dfrac{\ln^2 x.}{\sqrt{1+3\ln^2 x}}.\dfrac{\ln x dx}{x}$

Đặt $\sqrt{1+3\ln^2 x} = t \Rightarrow \ln^2 x = \dfrac{1}{3}(t^2 - 1) \Rightarrow \ln x.\dfrac{dx}{x} = \dfrac{1}{3}tdt$.

Suy ra $I = \dfrac{1}{9\ln^3 2}\left(\dfrac{1}{3}t^3 - t\right)\Big|_1^2 = \dfrac{4}{27\ln^3 2}$.

Câu 23. $I = \displaystyle\int_1^e \dfrac{x + (x-2)\ln x}{x(1+\ln x)}dx$

• $\displaystyle\int_1^e dx - 2\int_1^e \dfrac{\ln x}{x(1+\ln x)}dx = e - 1 - 2\int_1^e \dfrac{\ln x}{x(1+\ln x)}dx$

Tính $J = \displaystyle\int_1^e \dfrac{\ln x}{x(1+\ln x)}dx$. *Đặt* $t = 1 + \ln x \Rightarrow J = \displaystyle\int_1^2 \dfrac{t-1}{t}dt = 1 - \ln 2$.

Vậy: $I = e - 3 + 2\ln 2$.

Câu 24. $I = \displaystyle\int_{e^2}^{e^3} \dfrac{2x\ln^2 x - x\ln x^2 + 3}{x(1-\ln x)}dx$

• $I = 3\displaystyle\int_{e^2}^{e^3} \dfrac{1}{x(1-\ln x)}dx - 2\int_{e^2}^{e^3} \ln x dx = -3\ln 2 - 4e^3 + 2e^2$.

Câu 25. $I = \displaystyle\int_1^{e^2} \dfrac{\sqrt{\ln^2 x - \ln x^2 + 1}}{x^2}dx$

• *Đặt :* $t = \ln x \Rightarrow dt = \dfrac{dx}{x} \Rightarrow I = \int_0^2 \dfrac{\sqrt{t^2 - 2t + 1}}{e^t}dt = \int_0^2 \dfrac{|t-1|}{e^t}dt = -\int_0^1 \dfrac{t-1}{e^t}dt + \int_1^2 \dfrac{t-1}{e^t}dt = I_1 + I_2$

+ $I_1 = -\left(\int_0^1 \dfrac{tdt}{e^t} - \int_0^1 \dfrac{dt}{e^t}\right) = -\left(-te^{-t}\Big|_0^1 + \int_0^1 \dfrac{dt}{e^t} - \int_0^1 \dfrac{dt}{e^t}\right) = \dfrac{1}{e}$

+ $I_2 = \int_1^2 \dfrac{tdt}{e^t} - \int_1^2 \dfrac{dt}{e^t} = -te^{-t}\Big|_1^2 + \int_1^2 \dfrac{dt}{e^t} - \int_1^2 \dfrac{dt}{e^t} = -te^{-t}\Big|_1^2 = \dfrac{1}{e} - \dfrac{2}{e^2}$

Vậy : $I=\dfrac{2(e-1)}{e^2}$

Câu 26. $I=\int\limits_2^5 \dfrac{\ln(\sqrt{x-1}+1)}{x-1+\sqrt{x-1}}dx$

• *Đặt* $t=\ln\left(\sqrt{x-1}+1\right) \Rightarrow 2dt=\dfrac{dx}{x-1+\sqrt{x-1}} \Rightarrow I=2\int\limits_{\ln 2}^{\ln 3} dt=\ln^2 3-\ln^2 2$.

Câu 27. $I=\int\limits_1^{e^3} \dfrac{\ln^3 x}{x\sqrt{1+\ln x}}dx$

• *Đặt* $t=\sqrt{1+\ln x} \Rightarrow 1+\ln x=t^2 \Rightarrow \dfrac{dx}{x}=2tdt$ *và* $\ln^3 x=(t^2-1)^3$

$\Rightarrow I=\int\limits_1^2 \dfrac{(t^2-1)^3}{t}dt = \int\limits_1^2 \dfrac{t^6-3t^4+3t^2-1}{t}dt=\int\limits_1^2 (t^5-3t^3+3t-\dfrac{1}{t})dt=\dfrac{15}{4}-\ln 2$

Câu 28. $I=\int\limits_1^{\sqrt{e}} \dfrac{3-2\ln x}{x\sqrt{1+2\ln x}}dx$ • *Đặt* $t=\sqrt{1+2\ln x} \Rightarrow I=\int\limits_1^{\sqrt{e}} (2-t^2)dt=\dfrac{4\sqrt{2}-5}{3}$

Câu 29. $I=\int\limits_1^e \dfrac{\ln x\sqrt[3]{2+\ln^2 x}}{x}dx$ • *Đặt* $t=2+\ln^2 x \Rightarrow I=\dfrac{3}{8}\left[\sqrt[3]{3^4}-\sqrt[3]{2^4}\right]$

Câu 30. $I=\int\limits_1^e \dfrac{xe^x+1}{x(e^x+\ln x)}dx$ • *Đặt* $t=e^x+\ln x \Rightarrow I=\ln\dfrac{e^e+1}{e}$.

Dạng 2: Tích phân từng phần

Câu 31. $I = \int_0^{\frac{\pi}{2}} e^{\sin x}.\sin 2x dx$

• $I = 2\int_0^{\frac{\pi}{2}} e^{\sin x}.\sin x\cos x dx$. *Đặt* $\begin{cases} u = \sin x \\ dv = e^{\sin x}\cos x dx \end{cases} \Rightarrow \begin{cases} du = \cos x dx \\ v = e^{\sin x} \end{cases}$

$\Rightarrow I = 2\sin x e^{\sin x}\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^{\sin x}.\cos x dx = 2e - 2e^{\sin x}\Big|_0^{\frac{\pi}{2}} = 2$

Câu 32. $I = \int_0^1 x\ln(x^2+x+1)dx$

• *Đặt* $\begin{cases} u = \ln(x^2+x+1) \\ dv = xdx \end{cases} \Rightarrow \begin{cases} du = \dfrac{2x+1}{x^2+x+1}dx \\ v = \dfrac{x^2}{2} \end{cases}$

$I = \frac{x^2}{2}\ln(x^2+x+1)\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{2x^3+x^2}{x^2+x+1}dx$

$= \frac{1}{2}\ln 3 - \frac{1}{2}\int_0^1 (2x-1)dx + \frac{1}{4}\int_0^1 \frac{2x+1}{x^2+x+1}dx - \frac{3}{4}\int_0^1 \frac{dx}{x^2+x+1} = \frac{3}{4}\ln 3 - \frac{\sqrt{3}\pi}{12}$

Câu 33. $I = \int_3^8 \frac{\ln x}{\sqrt{x+1}}dx$

• *Đặt* $\begin{cases} u = \ln x \\ dv = \dfrac{dx}{\sqrt{x+1}} \end{cases} \Rightarrow \begin{cases} du = \dfrac{dx}{x} \\ v = 2\sqrt{x+1} \end{cases} \Rightarrow I = \left(2\sqrt{x+1}.\ln x\right)\Big|_3^8 - 2\int_3^8 \frac{\sqrt{x+1}}{x}dx = 6\ln 8 - 4\ln 3 - 2J$

+ *Tính* $J = \int_3^8 \frac{\sqrt{x+1}}{x}dx$. *Đặt* $t = \sqrt{x+1} \Rightarrow J = \int_2^3 \frac{t}{t^2-1}.2tdt = 2\int_2^3 \frac{t^2}{t^2-1}dt = \int_2^3 \left(2 + \frac{1}{t-1} - \frac{1}{t+1}\right)dt$

$= \left(2t + \ln\left|\frac{t-1}{t+1}\right|\right)\Big|_3^8 = 2 + \ln 3 - \ln 2$

Từ đó $I = 20\ln 2 - 6\ln 3 - 4$.

Câu 34. $I = \int_1^e \frac{x^2 + x\ln x + 1}{x}e^x dx$

• $I = \int_1^e xe^x dx + \int_1^e \ln x e^x dx + \int_1^e \frac{e^x}{x}dx$. + *Tính* $I_1 = \int_1^e xe^x dx = xe^x\Big|_1^e - \int_1^e e^x dx = e^e(e-1)$

+*Tính* $I_2 = \int_1^e e^x \ln x dx = e^x \ln x\Big|_1^e - \int_1^e \frac{e^x}{x}dx = e^e - \int_1^e \frac{e^x}{x}dx$.

Vậy: $I = I_1 + I_2 + \int_1^e \frac{e^x}{x}dx = e^{e+1}$.

Câu 35. $I=\int_1^e \left(\frac{\ln x}{x\sqrt{1+\ln x}}+\ln^2 x\right)dx$

• *Tính* $I_1=\int_1^e \frac{\ln x}{x\sqrt{1+\ln x}}dx$. *Đặt* $t=\sqrt{1+\ln x} \Rightarrow I_1=\frac{4}{3}-\frac{2\sqrt{2}}{3}$.

+ *Tính* $I_2=\int_1^e \ln^2 x dx$. *Lấy tích phân từng phần 2 lần được* $I_2=e-2$.

Vậy $I=e-\frac{2}{3}-\frac{2\sqrt{2}}{3}$.

Câu 36. $I=\int_1^2 \frac{\ln(x^2+1)}{x^3}dx$

• *Đặt* $\begin{cases} u=\ln(x^2+1) \\ dv=\frac{dx}{x^3} \end{cases} \Rightarrow \begin{cases} du=\frac{2x}{x^2+1} \\ v=-\frac{1}{2x^2} \end{cases}$. *Do đó* $I=-\frac{\ln(x^2+1)}{2x^2}\Big|_1^2+\int_1^2 \frac{dx}{x(x^2+1)}$

$=\frac{\ln 2}{2}-\frac{\ln 5}{8}+\int_1^2\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx=\frac{\ln 2}{2}-\frac{\ln 5}{8}+\int_1^2\frac{dx}{x}-\frac{1}{2}\int_1^2\frac{d(x^2+1)}{x^2+1}$

$=\frac{\ln 2}{2}-\frac{\ln 5}{8}+\left(\ln|x|-\frac{1}{2}\ln|x^2+1|\right)\Big|_1^2=2\ln 2-\frac{5}{8}\ln 5$

Câu 37. $I=\int_1^2 \frac{\ln(x+1)}{x^2}dx$

• *Đặt* $\begin{cases} u=\ln(x+1) \\ dv=\frac{dx}{x^2} \end{cases} \Leftrightarrow \begin{cases} du=\frac{dx}{x+1} \\ v=-\frac{1}{x} \end{cases} \Rightarrow I=-\frac{1}{x}\ln(x+1)\Big|_1^2+\int_1^2\frac{dx}{(x+1)x}=3\ln 2-\frac{3}{2}\ln 3$

Câu 38. $I=\int_0^{\frac{1}{2}} x\ln\left(\frac{1+x}{1-x}\right)dx$

• *Đặt* $\begin{cases} u=\ln\frac{1+x}{1-x} \\ dv=xdx \end{cases} \Rightarrow \begin{cases} du=\frac{2}{(1-x)^2}dx \\ v=\frac{x^2}{2} \end{cases} \Rightarrow I=\frac{1}{2}\left[x^2\ln\left(\frac{1+x}{1-x}\right)\Big|_0^{\frac{1}{2}}-\int_0^{\frac{1}{2}}x^2\left(\frac{2}{1-x^2}\right)dx\right]$

$=\frac{\ln 3}{8}+\int_0^{\frac{1}{2}}\frac{x^2}{x^2-1}dx=\frac{\ln 3}{8}+\int_0^{\frac{1}{2}}\left[1+\frac{1}{(x-1)(x+1)}\right]dx=\frac{\ln 3}{8}+\frac{1}{2}+\frac{1}{2}\ln\frac{2}{3}$

Câu 39. $I=\int_1^2 x^2.\ln\left(x+\frac{1}{x}\right)dx$ • *Đặt* $\begin{cases} u=\ln\left(x+\frac{1}{x}\right) \\ dv=x^2dx \end{cases} \Rightarrow I=3\ln 3-\frac{10}{3}\ln 2+\frac{1}{6}$

Câu 40. $I=\int_0^1 x^2.\ln(1+x^2)dx$ • *Đặt* $\begin{cases} u=\ln(1+x^2) \\ dv=x^2dx \end{cases} \Rightarrow I=\frac{1}{3}.\ln 2+\frac{4}{9}+\frac{\pi}{6}$

Câu 41. $I=\int_1^3 \frac{\ln x}{(x+1)^2}dx$ • *Đặt* $\begin{cases} u=\ln x \\ dv=\frac{dx}{(x+1)^2}\end{cases} \Rightarrow \quad I=-\frac{1}{4}\ln 3+\ln\frac{3}{2}$

Câu 42. $I=\int_1^e \frac{\ln^2 x+e^x(e^x+\ln^2 x)}{1+e^x}.dx$

• *Ta có:* $I=\int_1^e \ln^2 x.dx+\int_1^e \frac{e^{2x}}{e^x+1}dx=H+K$

+ $H=\int_1^e \ln^2 x.dx$. *Đặt:* $\begin{cases} u=\ln^2 x \\ dv=dx\end{cases} \Rightarrow H=e-\int_1^e 2\ln x.dx=e-2$

+ $K=\int_1^e \frac{e^{2x}}{e^x+1}dx$. *Đặt* $t=e^x+1 \Rightarrow \Rightarrow I_2=\int_{e+1}^{e^e+1}\frac{t-1}{t}dt=e^e-e+\ln\frac{e+1}{e^e+1}$

Vậy: $I=e^e-2+\ln\frac{e+1}{e^e+1}$

Câu 43. $I=\int_{\frac{1}{2}}^{2}(x+1-\frac{1}{x})e^{x+\frac{1}{x}}dx$

• *Ta có:* $I=\int_{\frac{1}{2}}^{2} e^{x+\frac{1}{x}}dx+\int_{\frac{1}{2}}^{3}\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx=H+K$

+ *Tính H theo phương pháp từng phần* $I_1 = H=xe^{x+\frac{1}{x}}\Big|_{\frac{1}{2}}^{2}-\int_{\frac{1}{2}}^{2}\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx=\frac{3}{2}e^{\frac{5}{2}}-K$

$\Rightarrow I=\frac{3}{2}e^{\frac{5}{2}}.$

Câu 44. $I=\int_0^4 \ln(\sqrt{x^2+9}-x)dx$

• *Đặt* $\begin{cases} u=\ln\left(\sqrt{x^2+9}-x\right) \\ dv=dx\end{cases} \Rightarrow I=x\ln\left(\sqrt{x^2+9}-x\right)\Big|_0^4+\int_0^4\frac{x}{\sqrt{x^2+9}}dx=2$

TP5: TÍCH PHÂN TỔ HỢP NHIỀU HÀM SỐ

Câu 1. $I=\int_0^1\left(x^2e^{x^3}+\frac{\sqrt[4]{x}}{1+\sqrt{x}}\right)dx$

• $I=\int_0^1 x^2e^{x^3}dx+\int_0^1\frac{\sqrt[4]{x}}{1+\sqrt{x}}dx$.

+ *Tính* $I_1=\int_0^1 x^2e^{x^3}dx$. *Đặt* $t=x^3$ $\Rightarrow I_1=\frac{1}{3}\int_0^1 e^t dt=\frac{1}{3}e^t\Big|_0^1=\frac{1}{3}e-\frac{1}{3}$.

+ *Tính* $I_2=\int_0^1\frac{\sqrt[4]{x}}{1+\sqrt{x}}dx$. *Đặt* $t=\sqrt[4]{x}$ $\Rightarrow I_2=4\int_0^1\frac{t^4}{1+t^2}dt=4\left(-\frac{2}{3}+\frac{\pi}{4}\right)$

Vậy: $I=\frac{1}{3}e+\pi-3$

Câu 2. $I=\int_1^2 x\left(e^x-\frac{\sqrt{4-x^2}}{x^3}\right)dx$

• $I=\int_1^2 xe^x dx+\int_1^2\frac{\sqrt{4-x^2}}{x^2}dx$.

+ *Tính* $I_1=\int_1^2 xe^x dx=e^2$ + *Tính* $I_2=\int_1^2\frac{\sqrt{4-x^2}}{x^2}dx$. *Đặt* $x=2\sin t$, $t\in\left[0;\frac{\pi}{2}\right]$.

$\Rightarrow I_2=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos^2 t}{\sin^2 t}dt=(-\cot t-t)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\sqrt{3}-\frac{\pi}{3}$

Vậy: $I=e^2+\sqrt{3}-\frac{\pi}{3}$.

Câu 3. $I=\int_0^1\frac{x}{\sqrt{4-x^2}}\left(e^{2x}.\sqrt{4-x^2}-x^2\right)dx$.

• $I=\int_0^1 x e^{2x}dx-\int_0^1\frac{x^3}{\sqrt{4-x^2}}dx=I_1+I_2$

+ *Tính* $I_1=\int_0^1 x e^{2x}dx=\frac{e^2+1}{4}$

+ *Tính* $I_2=\int_0^1\frac{x^3}{\sqrt{4-x^2}}dx$. *Đặt* $t=\sqrt{4-x^2}$ $\Rightarrow I_2=-3\sqrt{3}+\frac{16}{3}$

$\Rightarrow I=\frac{e^2}{4}+3\sqrt{3}-\frac{61}{12}$

Câu 4. $I=\int_0^1\frac{x^2+1}{(x+1)^2}e^x dx$

• *Đặt* $t = x+1 \Rightarrow dx = dt$ $I = \int_1^2 \frac{t^2-2t+2}{t^2}e^{t-1}dt = \int_1^2\left(1+\frac{2}{t^2}-\frac{2}{t}\right)e^{t-1}dt = e-1+\frac{2}{e}\left(-\frac{e^2}{2}+e\right)=1$

Câu 5. $I = \int_0^{\sqrt{3}} \frac{x^3.e^{\sqrt{x^2+1}}dx}{\sqrt{1+x^2}}$

• *Đặt* $t = \sqrt{1+x^2} \Rightarrow dx = tdt \Rightarrow I = \int_1^2 (t^2-1)e^t dt = \int_1^2 t^2e^tdt - e^t\Big|_1^2 = J-(e^2-e)$

$+\quad J = \int_1^2 t^2e^tdt = t^2e^t\Big|_1^2 - \int_1^2 2te^tdt = 4e^2-e-2\left(te^t\Big|_1^2-\int_1^2 e^tdt\right) = 4e^2-e-2(te^t-e^t)\Big|_1^2$

Vậy: $I = e^2$

Câu 6. $I = \int \frac{x\ln(x^2+1)+x^3}{x^2+1}dx$

• *Ta có:* $f(x) = \frac{x\ln(x^2+1)}{x^2+1}+\frac{x(x^2+1)-x}{x^2+1} = \frac{x\ln(x^2+1)}{x^2+1}+x-\frac{x}{x^2+1}$

$\Rightarrow F(x) = \int f(x)dx = \frac{1}{2}\int \ln(x^2+1)d(x^2+1)+\int xdx - \frac{1}{2}\int d\ln(x^2+1)$

$= \frac{1}{4}\ln^2(x^2+1)+\frac{1}{2}x^2-\frac{1}{2}\ln(x^2+1)+C.$

Câu 7. $I = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx$

• $I = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)}{\sqrt{x^2+9}}dx - 3\int_0^4 \frac{x^3}{\sqrt{x^2+9}}dx = I_1-3I_2$

\+ *Tính* $I_1 = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)}{\sqrt{x^2+9}}dx$. *Đặt* $\ln\left(x+\sqrt{x^2+9}\right) = u \Rightarrow du = \frac{1}{\sqrt{x^2+9}}dx$

$\Rightarrow I_1 = \int_{\ln 3}^{\ln 5} udu = \frac{u^2}{2}\Big|_{\ln 3}^{\ln 5} = \frac{\ln^2 5-\ln^2 3}{2}$

\+ *Tính* $I_2 = \int_0^4 \frac{x^3}{\sqrt{x^2+9}}dx$. *Đặt* $\sqrt{x^2+9} = v \Rightarrow dv = \frac{x}{\sqrt{x^2+9}}dx,\ \ x^2 = v^2-9$

$\Rightarrow I_2 = \int_3^5 (u^2-9)du = (\frac{u^3}{3}-9u)\Big|_3^5 = \frac{44}{3}$

Vậy $I = \int_0^4 \frac{\ln\left(x+\sqrt{x^2+9}\right)-3x^3}{\sqrt{x^2+9}}dx = I_1-3I_2 = \frac{\ln^2 5-\ln^2 3}{2}-44.$

Câu 8. $I = \int_1^e \frac{(x^3+1)\ln x+2x^2+1}{2+x\ln x}dx$

• $I = \int_1^e x^2dx + \int_1^e \frac{1+\ln x}{2+x\ln x}dx.$ $\quad + \int_1^e x^2dx = \frac{x^3}{3}\Big|_1^e = \frac{e^3-1}{3}$

$$+\int_1^e \frac{1+\ln x}{2+x\ln x}dx = \int_1^e \frac{d(2+x\ln x)}{2+x\ln x} = \ln|2+x\ln x|\Big|_1^e = \ln\frac{e+2}{2}.$$ *Vậy:* $I = \frac{e^3-1}{3}+\ln\frac{e+2}{2}$.

Câu 9. $I = \int_0^{\frac{\pi}{2}} \frac{1+\sin x}{1+\cos x}.e^x dx$

• $I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{e^x dx}{\cos^2\frac{x}{2}} + \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}e^x dx$

\+ *Tính* $I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}e^x dx = \int_0^{\frac{\pi}{2}} \frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}e^x dx = \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}e^x dx$

\+ *Tính* $I_2 = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{e^x dx}{\cos^2\frac{x}{2}}$. *Đặt* $\begin{cases} u = e^x \\ dv = \dfrac{dx}{2\cos^2\frac{x}{2}} \end{cases} \Rightarrow \begin{cases} du = e^x dx \\ v = \tan\dfrac{x}{2} \end{cases} \Rightarrow I_2 = e^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}e^x dx$

Do đó: $I = I_1 + I_2 = e^{\frac{\pi}{2}}$.

Câu 10. $I = \int_0^{\frac{\pi}{4}} \frac{\tan x.\ln(\cos x)}{\cos x}dx$

• *Đặt* $t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow I = -\int_1^{\frac{1}{\sqrt{2}}} \frac{\ln t}{t^2}dt = \int_{\frac{1}{\sqrt{2}}}^1 \frac{\ln t}{t^2}dt$.

Đặt $\begin{cases} u = \ln t \\ dv = \dfrac{1}{t^2}dt \end{cases} \Rightarrow \begin{cases} du = \dfrac{1}{t}dt \\ v = -\dfrac{1}{t} \end{cases} \Rightarrow I = \sqrt{2}-1-\frac{\sqrt{2}}{2}\ln 2$

Câu 11. $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{e^x(1+\sin 2x)}dx$

• $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{e^x(\sin x+\cos x)^2}dx$. *Đặt* $\begin{cases} u = \dfrac{\cos x}{e^x} \\ dv = \dfrac{dx}{(\sin x+\cos x)^2} \end{cases} \Rightarrow \begin{cases} du = \dfrac{-(\sin x+\cos x)dx}{e^x} \\ v = \dfrac{\sin x}{\sin x+\cos x} \end{cases}$

$\Rightarrow I = \frac{\cos x}{e^x}.\frac{\sin x}{\sin x+\cos x}\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{e^x} = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{e^x}$

Đặt $\begin{cases} u_1 = \sin x \\ dv_1 = \dfrac{dx}{e^x} \end{cases} \Rightarrow \begin{cases} du_1 = \cos x dx \\ v_1 = \dfrac{-1}{e^x} \end{cases} \Rightarrow I = \sin x.\frac{-1}{e^x}\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{e^x} = \frac{-1}{e^{\frac{\pi}{2}}} + \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{e^x}$

Đặt $\begin{cases} u_2 = \cos x \\ dv_1 = \dfrac{dx}{e^x} \end{cases} \Rightarrow \begin{cases} du_2 = -\sin x dx \\ v_1 = \dfrac{-1}{e^x} \end{cases}$

$$\Rightarrow I = \frac{-1}{e^{\frac{\pi}{2}}} + \cos x.\frac{-1}{e^x}\Bigg|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{e^x} = \frac{-1}{e^{\frac{\pi}{2}}} + 1 - I \Rightarrow 2I = -e^{\frac{-\pi}{2}} + 1 \Rightarrow I = \frac{-e^{\frac{-\pi}{2}}}{2} + \frac{1}{2}$$

Câu 12. $I = \int_0^{\frac{\pi}{2}} \sin x \ln(1+\sin x) dx$

• *Đặt* $\begin{cases} u = \ln(1+\sin x) \\ dv = \sin x dx \end{cases} \Rightarrow \begin{cases} du = \dfrac{1+\cos x}{1+\sin x} dx \\ v = -\cos x \end{cases}$

$$\Rightarrow I = -\cos x.\ln(1+\sin x)\Bigg|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x.\frac{\cos x}{1+\sin x} dx = 0 + \int_0^{\frac{\pi}{2}} \frac{1-\sin^2 x}{1+\sin x} dx = \int_0^{\frac{\pi}{2}} (1-\sin x) dx = \frac{\pi}{2} - 1$$

Câu 13. $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^6 x + \cos^6 x}{6^x + 1} dx$

• *Đặt* $t = -x \Rightarrow dt = -dx \Rightarrow I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^t \frac{\sin^6 t + \cos^6 t}{6^t + 1} dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 6^x \frac{\sin^6 x + \cos^6 x}{6^x + 1} dx$

$$\Rightarrow 2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (6^x + 1)\frac{\sin^6 x + \cos^6 x}{6^x + 1} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin^6 x + \cos^6 x) dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{5}{8} + \frac{3}{8}\cos 4x\right) dx = \frac{5\pi}{16}$$

$\Rightarrow I = \dfrac{5\pi}{32}$.

Câu 14. $I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^4 x dx}{2^{-x} + 1}$

• *Ta có:* $I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{2^x \sin^4 x dx}{2^x + 1} = \int_{-\frac{\pi}{6}}^{0} \frac{2^x \sin^4 x dx}{2^x + 1} + \int_0^{\frac{\pi}{6}} \frac{2^x \sin^4 x dx}{2^x + 1} = I_1 + I_2$

\+ *Tính* $I_1 = \int_{-\frac{\pi}{6}}^{0} \frac{2^x \sin^4 x dx}{2^x + 1}$. *Đặt* $x = -t \Rightarrow I_1 = -\int_{\frac{\pi}{6}}^{0} \frac{2^{-t} \sin^4(-t)}{2^{-t} + 1} dt = \int_{\frac{\pi}{6}}^{0} \frac{\sin^4 t}{2^t + 1} dt = \int_{\frac{\pi}{6}}^{0} \frac{\sin^4 x}{2^x + 1} dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{6}} \frac{\sin^4 x dx}{2^x + 1} + \int_0^{\frac{\pi}{6}} \frac{2^x \sin^4 x dx}{2^x + 1} = \int_0^{\frac{\pi}{6}} \sin^4 x dx = \frac{1}{4}\int_0^{\frac{\pi}{6}} (1 - \cos 2x)^2 dx$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{6}} (3 - 4\cos 2x + \cos 4x)dx \ = \frac{4\pi - 7\sqrt{3}}{64}$$

Câu 15. $I = \int_1^{e^3} \frac{\ln^3 x}{x\sqrt{1+\ln x}}dx$

• *Đặt* $t = \sqrt{1+\ln x} \Rightarrow 1 + \ln x = t^2 \Rightarrow \frac{dx}{x} = 2tdt$ *và* $\ln^3 x = (t^2-1)^3$

$$\Rightarrow I = \int_1^2 \frac{(t^2-1)^3}{t}dt = \int_1^2 \frac{t^6 - 3t^4 + 3t^2 - 1}{t}dt = \int_1^2 (t^5 - 3t^3 + 3t - \frac{1}{t})dt = \frac{15}{4} - \ln 2$$

Câu 16. $I = \int_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^2 x}dx$

• *Đặt* $\begin{cases} u = x \\ dv = \frac{\sin x}{\cos^2 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{\cos x} \end{cases} \Rightarrow I = \left.\frac{x}{\cos x}\right|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \frac{\pi\sqrt{2}}{4} - \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x}$

$+\ I_1 = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos x} = \int_0^{\frac{\pi}{4}} \frac{\cos x dx}{1 - \sin^2 x}$. *Đặt* $t = \sin x \Rightarrow I_1 = \int_0^{\frac{\sqrt{2}}{2}} \frac{dt}{1-t^2} = \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$

Vậy: $= \frac{\pi\sqrt{2}}{4} - \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}}$

Câu 17. $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{x\cos x}{\sin^3 x}dx$

• *Ta có* $\left(\frac{1}{\sin^2 x}\right)' = -\frac{2\cos x}{\sin^3 x}$. *Đặt* $\begin{cases} u = x \\ dv = \frac{\cos x}{\sin^3 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\frac{1}{2\sin^2 x} \end{cases}$

$$\Rightarrow I = -\frac{1}{2}x.\left.\frac{1}{\sin^2 x}\right|_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\sin^2 x} = -\frac{1}{2}(\frac{\pi}{2} - \frac{\pi}{2}) - \frac{1}{2}\cot x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2}.$$

Câu 18. $I = \int_0^{\frac{\pi}{4}} \frac{x\sin x}{\cos^3 x}dx$

• *Đặt:* $\begin{cases} u = x \\ dv = \frac{\sin x}{\cos^3 x}dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{2.\cos^2 x} \end{cases} \Rightarrow I = \left.\frac{x}{2\cos^2 x}\right|_0^{\frac{\pi}{4}} - \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x} = \frac{\pi}{4} - \frac{1}{2}\tan x\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{4} - \frac{1}{2}$

Câu 19. $I = \int_1^{e^\pi} \cos(\ln x)dx$

• *Đặt* $t = \ln x \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$\Rightarrow I = \int_0^{\pi} e^t \cos t dt = -\frac{1}{2}(e^{\pi} + 1)$ *(dùng pp tích phân từng phần).*

Câu 20. $I = \int_0^{\frac{\pi}{2}} e^{\sin^2 x}.\sin x.\cos^3 x dx$

• *Đặt* $t = \sin^2 x \Rightarrow I = \frac{1}{2}\int_0^1 e^t(1-t)dt = \frac{1}{2}e$ *(dùng tích phân từng phần)*

Câu 21. $I = \int_0^{\frac{\pi}{4}} \ln(1+\tan x)dx$

• *Đặt* $t = \frac{\pi}{4} - x \Rightarrow I = \int_0^{\frac{\pi}{4}} \ln\left(1+\tan\left(\frac{\pi}{4}-t\right)\right)dt = \int_0^{\frac{\pi}{4}} \ln\left(1+\frac{1-\tan t}{1+\tan t}\right)dt = \int_0^{\frac{\pi}{4}} \ln\frac{2}{1+\tan t}dt$

$= \int_0^{\frac{\pi}{4}} \ln 2 dt - \int_0^{\frac{\pi}{4}} \ln(1+\tan t)dt = t.\ln 2\Big|_0^{\frac{\pi}{4}} - I$

$\Rightarrow 2I = \frac{\pi}{4}\ln 2 \Rightarrow I = \frac{\pi}{8}\ln 2.$

Câu 22. $I = \int_1^4 \frac{\ln(5-x) + x^3.\sqrt{5-x}}{x^2}dx$

• *Ta có:* $I = \int_1^4 \frac{\ln(5-x)}{x^2}dx + \int_1^4 x\sqrt{5-x}.dx = K + H.$

$+\ K = \int_1^4 \frac{\ln(5-x)}{x^2}dx$. *Đặt* $\begin{cases} u = \ln(5-x) \\ dv = \frac{dx}{x^2} \end{cases} \Rightarrow K = \frac{3}{5}\ln 4$

$+\ H = \int_1^4 x\sqrt{5-x}.dx$. *Đặt* $t = \sqrt{5-x} \Rightarrow H = \frac{164}{15}$

Vậy: $I = \frac{3}{5}\ln 4 + \frac{164}{15}$

Câu 23. $I = \int_0^{\frac{\pi}{2}} \frac{(x+\sin^2 x)}{1+\sin 2x}dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{2}} \frac{x}{1+\sin 2x}dx + \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x}dx = H + K$

$+\ H = \int_0^{\frac{\pi}{2}} \frac{x}{1+\sin 2x}dx = \int_0^{\frac{\pi}{2}} \frac{x}{2\cos^2\left(x-\frac{\pi}{4}\right)}dx$. *Đặt:* $\begin{cases} u = x \\ dv = \frac{dx}{2\cos^2\left(x-\frac{\pi}{4}\right)} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \frac{1}{2}\tan\left(x-\frac{\pi}{4}\right) \end{cases}$

$\Rightarrow H = \frac{x}{2}\tan\left(x-\frac{\pi}{4}\right)\Big|_0^{\frac{\pi}{2}} + \left(\frac{1}{2}\ln\left|\cos\left(x-\frac{\pi}{4}\right)\right|\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$

+ $K = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x} dx$. *Đặt* $t = \frac{\pi}{2} - x \Rightarrow K = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1+\sin 2x} dx$

$$\Rightarrow 2K = \int_0^{\frac{\pi}{2}} \frac{dx}{2\cos^2\left(x - \frac{\pi}{4}\right)} = \frac{1}{2}\tan\left(x - \frac{\pi}{4}\right)\Big|_0^{\frac{\pi}{2}} = 1 \Rightarrow K = \frac{1}{2}$$

Vậy, $I = H + K = \frac{\pi}{4} + \frac{1}{2}$.

Câu 24. $I = \int_0^{\pi} \frac{x(\cos^3 x + \cos x + \sin x)}{1+\cos^2 x} dx$

• *Ta có:* $I = \int_0^{\pi} x\left(\frac{\cos x(1+\cos^2 x) + \sin x}{1+\cos^2 x}\right) dx = \int_0^{\pi} x.\cos x.dx + \int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x} dx = J + K$

+ *Tính* $J = \int_0^{\pi} x.\cos x.dx$. *Đặt* $\begin{cases} u = x \\ dv = \cos x dx \end{cases} \Rightarrow J = (x.\sin x)\Big|_0^{\pi} - \int_0^{\pi} \sin x.dx = 0 + \cos x\Big|_0^{\pi} = -2$

+ *Tính* $K = \int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x} dx$. *Đặt* $x = \pi - t \Rightarrow dx = -dt$

$$\Rightarrow K = \int_0^{\pi} \frac{(\pi - t).\sin(\pi - t)}{1+\cos^2(\pi - t)} dt = \int_0^{\pi} \frac{(\pi - t).\sin t}{1+\cos^2 t} dt = \int_0^{\pi} \frac{(\pi - x).\sin x}{1+\cos^2 x} dx$$

$$\Rightarrow 2K = \int_0^{\pi} \frac{(x + \pi - x).\sin x}{1+\cos^2 x} dx = \pi\int_0^{\pi} \frac{\sin x.dx}{1+\cos^2 x} \Rightarrow K = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x.dx}{1+\cos^2 x}$$

Đặt $t = \cos x \Rightarrow dt = -\sin x.dx \Rightarrow K = \frac{\pi}{2}\int_{-1}^{1} \frac{dt}{1+t^2}$, *đặt* $t = \tan u \Rightarrow dt = (1+\tan^2 u)du$

$$\Rightarrow K = \frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+\tan^2 u)du}{1+\tan^2 u} = \frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} du = \frac{\pi}{2}.u\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi^2}{4}$$

Vậy $I = \frac{\pi^2}{4} - 2$

Câu 25. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x + (x + \sin x)\sin x}{(1+\sin x)\sin^2 x} dx$

• *Ta có:* $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x(1+\sin x) + \sin^2 x}{(1+\sin x)\sin^2 x} dx = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x} = H + K$

+ $H = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x} dx$. *Đặt* $\begin{cases} u = x \\ dv = \frac{dx}{\sin^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\cot x \end{cases} \Rightarrow H = \frac{\pi}{\sqrt{3}}$

+ $K = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\cos\left(\frac{\pi}{2} - x\right)} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)} = \sqrt{3} - 2$

Vậy $I = \frac{\pi}{\sqrt{3}} + \sqrt{3} - 2$

Câu 26. $I = \int_0^2 \left[\sqrt{x(2-x)} + \ln(4+x^2) \right] dx$

• *Ta có:* $I = \int_0^2 \sqrt{x(2-x)}dx + \int_0^2 \ln(4+x^2)dx = I_1 + I_2$

\+ $I_1 = \int_0^2 \sqrt{x(2-x)}dx = \int_0^2 \sqrt{1-(x-1)^2}dx = \frac{\pi}{2}$ *(sử dụng đổi biến:* $x = 1 + \sin t$ *)*

\+ $I_2 = \int_0^2 \ln(4+x^2)dx = x\ln(4+x^2)\Big|_0^2 - 2\int_0^2 \frac{x^2}{4+x^2}dx$ *(sử dụng tích phân từng phần)*

$= 6\ln 2 + \pi - 4$ *(đổi biến* $x = 2\tan t$ *)*

Vậy: $I = I_1 + I_2 = \frac{3\pi}{2} - 4 + 6\ln 2$

Câu 27. $I = \int_0^{\frac{\pi}{3}} \frac{x + \sin^2 x}{1 + \cos 2x} dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{3}} \frac{x + \sin^2 x}{1 + \cos 2x} dx = \int_0^{\frac{\pi}{3}} \frac{x}{2\cos^2 x} dx + \int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{2\cos^2 x} dx = H + K$

\+ $H = \int_0^{\frac{\pi}{3}} \frac{x}{2\cos^2 x} dx = \frac{1}{2}\int_0^{\frac{\pi}{3}} \frac{x}{\cos^2 x} dx$. *Đặt* $\begin{cases} u = x \\ dv = \frac{dx}{\cos^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \tan x \end{cases}$

$\Rightarrow H = \frac{1}{2}\left[x\tan x\Big|_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} \tan x dx \right] = \frac{\pi}{2\sqrt{3}} + \frac{1}{2}\ln\left|\cos x\right|\Big|_0^{\frac{\pi}{3}} = \frac{\pi}{2\sqrt{3}} - \frac{1}{2}\ln 2$

\+ $K = \int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{2\cos^2 x} dx = \frac{1}{2}\int_0^{\frac{\pi}{3}} \tan^2 x dx = \frac{1}{2}\left[\tan x - x\right]\Big|_0^{\frac{\pi}{3}} = \frac{1}{2}\left(\sqrt{3} - \frac{\pi}{3} \right)$

Vậy: $I = H + K = \frac{\pi}{2\sqrt{3}} - \frac{1}{2}\ln 2 + \frac{1}{2}\left(\sqrt{3} - \frac{\pi}{3} \right) = \frac{\pi\left(\sqrt{3}-1\right)}{6} + \frac{1}{2}(\sqrt{3} - \ln 2)$

Câu 28. $I = \int_3^8 \frac{\ln x}{\sqrt{x+1}} dx$

• *Đặt* $\begin{cases} u = \ln x \\ dv = \frac{dx}{\sqrt{x+1}} \end{cases} \Rightarrow \begin{cases} du = \frac{dx}{x} \\ v = 2\sqrt{x+1} \end{cases} \Rightarrow I = 2\sqrt{x+1}\ln x\Big|_3^8 - 2\int_3^8 \frac{\sqrt{x+1}}{x}dx$

\+ *Tính* $J = \int_3^8 \frac{\sqrt{x+1}}{x}dx$. *Đặt* $t = \sqrt{x+1} \Rightarrow J = \int_2^3 \frac{2t^2 dt}{t^2 - 1} = 2\int_2^3 \left(1 + \frac{1}{t^2 - 1} \right) dt = 2 + \ln 3 - \ln 2$

$\Rightarrow I = 6\ln 8 - 4\ln 3 - 2(2 + \ln 3 - \ln 2) = 20\ln 2 - 6\ln 3 - 4$

Câu 29. $I = \int_1^2 \frac{1 + x^2}{x^3} \ln x dx$

• *Ta có:* $I = \int_1^2 \left(\frac{1}{x^3} + \frac{1}{x} \right) \ln x dx$. *Đặt* $\begin{cases} u = \ln x \\ dv = (\frac{1}{x^3} + \frac{1}{x})dx \end{cases}$

$$\Rightarrow I = \left(\frac{-1}{4x^4} + \ln x\right)\ln x\Big|_1^2 - \int_1^2 \left(\frac{-1}{4x^5} + \frac{1}{x}\ln x\right)dx \;=\; -\frac{1}{64}\ln 2 + \frac{63}{4} + \frac{1}{2}\ln^2 2$$

Câu 30. $I = \int_0^3 \sqrt{x+1}\sin\sqrt{x+1}.dx$

• *Đặt* $t = \sqrt{x+1} \Rightarrow I = \int_1^2 t.\sin t.2tdt = \int_1^2 2t^2 \sin tdt = \int_1^2 2x^2 \sin xdx$

Đặt $\begin{cases} u = 2x^2 \\ dv = \sin xdx \end{cases} \Rightarrow \begin{cases} du = 4xdx \\ v = -\cos x \end{cases} \Rightarrow I = -2x^2\cos x\Big|_1^2 + \int_1^2 4x\cos xdx$

Đặt $\begin{cases} u = 4x \\ dv = \cos xdx \end{cases} \Rightarrow \begin{cases} du = 4dx \\ v = \sin x \end{cases}$. *Từ đó suy ra kết quả.*

Câu 31. $I = \int_1^e \frac{x^2 + x\ln x + 1}{x} e^x dx$

• *Ta có:* $I = \int_1^e xe^x dx + \int_1^e e^x \ln xdx + \int_1^e \frac{e^x}{x}dx = H + K + J$

$+\; H = \int_1^e xe^x dx = xe^x\Big|_1^e - \int_1^e e^x dx = e^e(e-1)$

$+\; K = \int_1^e e^x \ln xdx = e^x \ln x\Big|_1^e - \int_1^e \frac{e^x}{x}dx = e^e - \int_1^e \frac{e^x}{x}dx = e^e - J$

Vậy: $I = H + K + J = e^{e+1} - e^e + e^e - J + J = e^{e+1}$.

TP6: TÍCH PHÂN HÀM SỐ ĐẶC BIỆT

Câu 1. Cho hàm số *f(x)* liên tục trên R và $f(x)+f(-x)=\cos^4 x$ với mọi $x\in$ R.

Tính: $I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)dx$.

• *Đặt* $x=-t \Rightarrow \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)dx=\int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} f(-t)(-dt)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(-t)dt=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(-x)dx$

$\Rightarrow 2\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[f(x)+f(-x)\right]dx=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\cos^4 xdx \Rightarrow I=\frac{3\pi}{16}$

Chú ý: $\cos^4 x=\frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x$.

Câu 2. Cho hàm số *f(x)* liên tục trên R và $f(x)+f(-x)=\sqrt{2+2\cos 2x}$, với mọi $x\in$ R.

Tính: $I=\int_{\frac{-3\pi}{2}}^{\frac{3\pi}{2}} f(x)dx$.

• *Ta có :* $I=\int_{-3\frac{\pi}{2}}^{3\frac{\pi}{2}} f(x)dx=\int_{-3\frac{\pi}{2}}^{0} f(x)dx+\int_{0}^{3\frac{\pi}{2}} f(x)dx$ *(1)*

+ *Tính :* $I_1=\int_{-3\frac{\pi}{2}}^{0} f(x)dx$. *Đặt* $x=-t\Rightarrow dx=-dt\Rightarrow I_1=\int_{0}^{3\frac{\pi}{2}} f(-t)dt=\int_{0}^{3\frac{\pi}{2}} f(-x)dx$

Thay vào (1) ta được: $I=\int_{0}^{3\frac{\pi}{2}}\left[f(-x)+f(x)\right]dx=\int_{0}^{3\frac{\pi}{2}}\sqrt{2\left(1+\cos 2x\right)}=2\int_{0}^{3\frac{\pi}{2}}\left|\cos x\right|dx$

$=2\left[\int_{0}^{\frac{\pi}{2}}\cos xdx-\int_{\frac{\pi}{2}}^{3\frac{\pi}{2}}\cos xdx\right]=2\left[\sin x\Big|_0^{\frac{\pi}{2}}-\sin x\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right]=6$

Câu 3. $I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sin x}{\sqrt{1+x^2}+x}dx$

• $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1+x^2}\sin x dx - \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x\sin x dx = I_1 - I_2$

+ *Tính* $I_1 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1+x^2}\sin x dx$. *Sử dụng cách tính tích phân của hàm số lẻ, ta tính được* $I_1 = 0$.

+ *Tính* $I_2 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x\sin x dx$. *Dùng pp tích phân từng phần, ta tính được:* $I_2 = -\frac{\sqrt{2}}{4}\pi + \sqrt{2}$

Suy ra: $I = \frac{\sqrt{2}}{4}\pi - \sqrt{2}$.

Câu 4. $I = \int_2^5 \frac{e^x(3x-2)+\sqrt{x-1}}{e^x(x-1)+\sqrt{x-1}}dx$

• $I = \int_2^5 \frac{e^x(3x-2)+\sqrt{x-1}}{e^x(x-1)+\sqrt{x-1}}dx = \int_2^5 \frac{e^x(x-1)+\sqrt{x-1}+e^x(2x-1)}{e^x(x-1)+\sqrt{x-1}}dx = \int_2^5 dx + \int_2^5 \frac{e^x(2x-1)}{e^x(x-1)+\sqrt{x-1}}dx$

$= x\Big|_2^5 + \int_2^5 \frac{e^x(2x-1)}{\sqrt{x-1}(e^x\sqrt{x-1}+1)}dx = 3 + \int_2^5 \frac{e^x(2x-1)}{\sqrt{x-1}(e^x\sqrt{x-1}+1)}dx$

Đặt $t = e^x\sqrt{x-1}+1 \Rightarrow dt = \frac{e^x(2x-1)}{2\sqrt{x-1}}dx$

$\Rightarrow I = 3 + \int_{e^2+1}^{2e^5+1} \frac{2}{t}dt \Rightarrow I = 3 + 2\ln|t|\Big|_{e^2+1}^{2e^5+1} = 3 + 2\ln\left|\frac{2e^5+1}{e^2+1}\right|$

Câu 5. $I = \int_0^{\frac{\pi}{4}} \frac{x^2}{(x\sin x+\cos x)^2}dx$.

• $I = \int_0^{\frac{\pi}{4}} \frac{x}{\cos x}.\frac{x\cos x}{(x\sin x+\cos x)^2}dx$. *Đặt* $\begin{cases} u = \frac{x}{\cos x} \\ dv = \frac{x\cos x}{(x\sin x+\cos x)^2}dx \end{cases} \Rightarrow \begin{cases} du = \frac{\cos x + x\sin x}{\cos^2 x}dx \\ v = \frac{-1}{x\sin x+\cos x} \end{cases}$

$\Rightarrow I = -\frac{x}{\cos x(x\sin x+\cos x)}\Big|_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}dx = \frac{4-\pi}{4+\pi}$.