

# BÀI TẬP NGUYÊN HÀM, TÍCH PHÂN.

(LAISAC sưu tầm và biên soạn, lời giải sẽ post lên sau)

I, TÌM NGUYÊN HÀM:  $\int x^x(1+\ln x)dx$  ;  $\int \frac{2^x 3^x}{9^x - 4^x} dx$  ;  $\int \frac{\cos x + \sin x}{\sqrt{2 - \sin 2x}} dx$  ;  $\int \frac{\ln 2x}{x \ln 4x} dx$  ;

$$\int \frac{2^x 3^x}{9^x - 4^x} dx ; \int \frac{dx}{\sin 2x - 2 \sin x} ; \int \frac{x^2 + 1}{x^4 - 3x^2 + 1} dx.$$

## ĐỔI BIẾN SỐ :

Dạng hữu tỷ, đa thức:  $\int_0^1 \frac{2x+3}{x^2-4x+4} dx$  ;  $\int_0^1 \frac{-x+1}{x^2-2x-5} dx$  ;  $\int_0^1 \frac{x^3}{x^2-4x+5} dx$  ;  $\int_0^1 \frac{5x-2}{(2x-1)^3} dx$  ;  $\int_0^2 \frac{x^4-x+1}{x^4+1} dx$  ;

$$\int_1^2 \frac{x^2-1}{x^4+1} dx. \int_0^1 \frac{x^2 dx}{\sqrt{x^2+4}} ; \int_0^1 \frac{1+x^4}{1+x^6} dx ; \int_0^1 \frac{dx}{(1+3x^2)^2} ; \int_0^1 x^5(1-x^3)^6 dx ; \int_0^1 x^2(1-x^8) dx ; \int_0^3 |x^2-3x+2| dx ;$$

$$\int_{-3}^5 (|x+2|-|x-2|) dx ;$$

Dạng Vô tỉ:  $\int_1^2 \frac{x dx}{1+\sqrt{x-1}}$  ;  $\int_0^1 \frac{\sqrt{x}-1}{\sqrt[3]{x+1}} dx$  ;  $\int_0^1 \frac{x^2 dx}{\sqrt{2x+1}}$  ;  $\int_0^4 \frac{x^2 dx}{\sqrt[7]{x\sqrt{x^2+9}}}$  ;  $\int_0^2 \frac{dx}{x\sqrt{x^2+1}}$  ;  $\int_1^2 x^2 \sqrt{4-x^2} dx$  ;  $\int \frac{\sqrt{2}}{2} \frac{dx}{x\sqrt{x^2-1}}$

$$\int_0^1 x^2 \sqrt{4-3x^2} dx ; \int_0^1 \frac{x^2 dx}{\sqrt{x^2+4}} ; \int_0^{\sqrt{2}} \frac{x^2 dx}{\sqrt{1-x^2}} ; \int_{\frac{2}{\sqrt{3}}}^{2\sqrt{2}} \frac{dx}{x\sqrt{x^2+1}} ; \int_{\frac{\sqrt{2}}{2}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}} ; \int_0^1 \frac{dx}{\sqrt{x^2-x+1}} ; \int_1^{\sqrt{3}} \frac{\sqrt{9+3x^2}}{x^2} dx ;$$

Dạng Lượng Giác:  $\int_0^{\frac{\pi}{2}} \cos^5 x dx$  ;  $\int_0^{\frac{\pi}{2}} \sin^4 x dx$  ;  $\int_0^{\frac{\pi}{4}} tg^3 x dx$  ;  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\sin x}$  ;  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\cos x}$  ;  $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 x}$  ;  $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x}$  ;  $\int_0^{\frac{\pi}{4}} \frac{dx}{tg^4 x}$  ;

$$\int_0^{\frac{\pi}{2}} \sin 3x \cos 5x dx ; \int_0^{\frac{\pi}{2}} \sin^2 2x \cos x dx ; \int_0^{\frac{\pi}{2}} \frac{\sin 3x}{1+\cos x} dx ; \int_0^{\frac{\pi}{2}} \frac{x+\sin 3x}{1+\cos x} dx ; \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x+\cos x} dx ; \int_0^{\frac{\pi}{4}} \frac{\cos x - \sin x}{\sqrt{2+\sin 2x}} dx ;$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2x + \sin x}{\sqrt{1+3\cos x}} dx ; \int_0^{\frac{\pi}{2}} \frac{\sin 3x}{1+\cos x} dx ; \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x+\cos x} dx ; \int_0^{\frac{\pi}{2}} \sqrt{1-\cos x} dx ; \int_0^{\frac{\pi}{2}} \sqrt{1+\sin x} dx ; \int_0^{\frac{\pi}{3}} \frac{2\sin 2x + 3\sin x}{\sqrt{6\cos x - 2}} dx$$

$$; \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{tgx}{\cos x \sqrt{1+\cos^2 x}} dx ; \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sqrt{\sin^5 x \cos^3 x}} ; \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x + 2\sin x + 3} ; \int_0^{\frac{\pi}{6}} \frac{tg^3 x}{\cos 2x} dx ; \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos^4 x - 3\cos^2 x + 3} dx ;$$

$$\int_0^{\frac{\pi}{4}} \frac{x \cos^4 x + tgx}{1+\cos 2x} dx ; \int_{-1}^1 \ln(\sqrt{x^2+a^2}+x) dx ;$$

Dạng Mũ, Logarit:  $\int_0^{\ln 2} \frac{dx}{\sqrt{e^x+1}}$  ;  $\int_0^{\ln 2} \sqrt{e^x-1} dx$  ;  $\int_0^{\ln 2} \frac{dx}{\sqrt{e^x+1}}$  ;  $\int_0^{\ln 2} \sqrt{e^x-1} dx$  ;  $\int_{-\pi}^{\pi} \frac{x^4}{2^x+1} dx$  ;  $\int_{-\pi}^{\pi} \frac{1+\cos 2x}{e^x+1} dx$  ;  $\int_{\ln 4}^{\ln 12} \sqrt{e^x-3} dx$

$$\int_1^e \frac{\ln x \sqrt{2+\ln^2 x}}{x} dx ; \int_1^e \frac{\ln x}{x(2+\ln x)^2} dx ;$$

**Đặc Biệt:**  $\int_0^{\frac{\pi}{4}} \ln(1+\tan x) dx$ ,  $\int_{-\pi}^{\pi} \frac{1+\cos 2x}{e^x+1} dx$ ,  $\int_{\frac{1}{e}}^e \frac{\ln x}{(x+1)^2} dx$ . (Đặt  $x=1/t$ ),  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^6 x + \cos^6 x}{6^x+1} dx$

## II. TỪNG PHẦN:

$$\int_0^{\frac{\pi}{2}} x \cos^2 x dx; \int_1^e \ln^3 x dx; \int_1^{e^2} \ln \sqrt{x} dx; \int_{01}^1 x \ln(1+x^3) dx; \int_1^e x \ln^2 x dx; \int_0^{\frac{\pi}{2}} x \sin x \cos 2x dx; \int_0^1 e^{\sqrt{3x+1}} dx,$$

$$\int_0^{\frac{\pi}{4}} \frac{x}{1+\cos 2x} dx; \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{x \sin x}{\cos^2 x} dx; \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\ln(\sin x)}{\cos^2 x} dx; \int_{\frac{1}{e}}^e \frac{\ln x}{(x+1)^2} dx; \int_0^{\frac{\pi}{4}} \frac{x^2 \sin 2x}{\cos^4 x} dx; \int_0^1 \frac{x e^x dx}{\sqrt{e^x+1}}; \int_1^{\sqrt{3}} \frac{x \ln x dx}{\sqrt{x^2+1}}; \int_0^1 \frac{\ln x+1}{(x+2)^2} dx;$$

$$\int_0^{\frac{\pi}{2}} \frac{x dx}{1+\sin 2x}; \int_{-1}^1 \ln(\sqrt{x^2+a^2}+x) dx; \int_1^e x \ln^2 x dx; \int_1^e \ln^3 x dx; \int_0^{\frac{\pi}{4}} \frac{x^2 \sin 2x}{\cos^4 x} dx; \int_0^1 \frac{x e^x dx}{\sqrt{e^x+1}}; \int_1^{\sqrt{3}} \frac{x \ln x dx}{\sqrt{x^2+1}};$$

$$\int_0^{\frac{\pi}{4}} \frac{x}{1+\cos 2x} dx; \int_0^{\frac{\pi}{2}} x \cos^2 x dx; \int_{01}^1 x \ln(1+x^3) dx; \int_0^{\frac{\pi}{2}} x \sin x \cos 2x dx; \int_0^{\frac{\pi}{2}} x \sin x \cos 3x dx; \int_e^{e^2} \frac{1+x^2 \ln x}{1+\cos x} dx;$$

$$\int_0^{\frac{\pi}{2}} (x+\sin 2x) e^x dx; \int_0^{\frac{\pi}{2}} \frac{1-\sin x}{(1+\cos x) e^x} dx; \int_0^1 e^{\sqrt{3x+1}} dx; \int_0^{\frac{\pi}{2}} \frac{x dx}{1+\sin 2x}; \int_{\ln 3}^{\ln 8} \frac{x e^x dx}{\sqrt{e^x+1}}; \int_1^e \left( \frac{\ln x}{x\sqrt{1+\ln x}} + \ln^2 x \right) dx$$

**TỔNG HỢP và LIÊN KẾT**  $\int_0^{\frac{\pi}{2}} (x+\sin 2x) e^x dx; \int_0^{\frac{\pi}{2}} \frac{x \sin x + \sin 2x}{4-\cos^2 x} dx; \int_e^{e^2} \frac{1+x^2 \ln x}{1+\cos x} dx; \int_0^{\frac{\pi}{2}} \frac{x+\sin 3x}{1+\cos x} dx$

$$\int_0^{\frac{\pi}{4}} \frac{x \cos^4 x + \tan x}{1+\cos 2x} dx; \int_1^e \left( \frac{\ln x}{x\sqrt{1+\ln x}} + \ln^2 x \right) dx; \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos^4 x - 3\cos^2 x + 3} dx; \int_0^{\frac{\pi}{2}} \frac{1+\sin x}{1+\cos x} e^x dx,$$

**Liên Kết:**  $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin x + \cos x} dx; \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx; \int_0^{\frac{\pi}{2}} \frac{x \sin x}{1+\cos^2 x} dx; \int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos^2 x} dx; \int_0^{\frac{\pi}{2}} x \sin x \cos^{2008} x dx,$

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x+\cos x}{4-\sin^2 x} dx; \int_0^{\frac{\pi}{2}} \frac{x \sin x}{1+\cos^2 x} dx; \int_e^{e^{\pi}} \cos(\ln x) dx; \int_e^{e^2} \left( \frac{1}{\ln^2 x} - \frac{1}{\ln x} \right) dx; \int_0^{\frac{\pi}{2}} \frac{x \sin x}{\cos^2 x - 4} dx; \int_0^1 \frac{2014^x}{2014^x + 2014^{-x}} dx$$

**Giải các phương trình:**  $\int_0^x \sin 2t \sqrt{1+\cos^2 t} dt = 0$ ;  $\int_0^x \frac{t^2 e^t}{(t+2)^2} dt = 1$  với  $x > 0$

**Bài của LAISAC (5 đề trong Thử sức trước kỳ thi báo Toán Học và Tuổi trẻ)**

$$\int_0^1 (2x^2+x+1)e^{x^2+x+1} dx; \int_0^{\frac{\pi}{2}} \left( 2\cos^2 \frac{x}{2} + x \cos x \right) e^{\sin x} dx; \int_1^e \ln \left( \sqrt{1+\ln^2 x} + \ln x \right)^{\frac{1}{x}} dx; I = \int_0^{\frac{\pi}{4}} e^x (ta^2 nx + \ln(\cos x)) dx,$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\ln(4 \tan x)}{\sin 2x \ln(2 \tan x)} dx$$